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Prediction Performance and Generalization of the Empirical Estimation of Rockmass Deformation Modulus Based on Rockmass Classification Systems

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Abstract: The deformation modulus of rock mass can either be measured using in-situ field tests or estimated empirically. However, the empirical method is an attractive and open choice for researchers to estimate the deformation modulus of rock mass economically, efficiently and accurately and many researchers have proposed empirical equations for estimation the deformation modulus of rock mass based rock mass classification systems. In this paper attempt has been made to collect and review some these published empirical equations developed so far based on Rock Mass Rating, Geological Strength Index, and Q rock mass classification systems. These empirical equations based on each classification system are grouped on the based on similar parameters. All the equations within certain group for all groups were plotted to obtain a viable range for which the equation is assumed to be reliable. An artificial data is generated for each group separately to from valid data points of existing equations. Keeping in view the trend of data points in the scatter plot of individual group, Gaussian and Sigmoidal type mathematical functions are fitted to obtain generalized equation for each group that consisted two or more than two valid equations. The new equations fitted to the data are optimized by minimizing residuals using Microsoft Excel built in Add-in "Solver". It is observed that equations using sigmoidal function predict more precisely than equations using Gaussian function.

Key Words: **Deformation modulus; Rockmass; Estimation methods; Empirical; Gaussian; Sigmoidal;**

I Introduction

The deformation modulus of rock mass is an important parameter and used as essential input used in many numerical tools codes for response of ground to stress after excavation of tunnels within rock mass. The parameter can be directly determined directly using different in situ test procedures however; it requires significant cost and involves tedious operation process. Furthermore the intrinsic errors and other associated problems provide a base to search for alternative methods to estimate the deformation modulus of rock mass indirectly and is an interesting issue for research in the field of rock engineering. Historically the deformation modulus of rock mass is estimated using either empirical method utilizing rock mass classification systems, rock quality and seismic velocity in rock mass or utilizing certain models as by Duncan and Goodman, 1968 and Kulhawy, 1978 (Benson, 1986).

Deere, 1969 made the first attempt to estimate the deformation modulus of rock mass establishing correlation between modulus obtained from in situ tests and Rock Quality Designation (RQD). The correlation was poor as there was great scatter in the data. Continuing the line Bieniawski, 1978 felt that the intact rock properties influenced the rock mass properties and developed an equation based RQD and modulus reduction factor which is the ratio between in situ modulus and laboratory modulus of rocks. Recently some new correlations have been developed based on either modulus ratio and RQD (Zhang & Einstein , 2004) and (Jing, 2003) correlating intact rock modulus E_i, intact rock strength σ_c, weathering degree WD of rock mass with RQD (Kayabasi, Gokceoglu, & Ercanoglu, 2003) and (Gokceoglu, Sonmez, & Kayabasi , 2003). Some of equations based on such parameters are listed in Table 1. However, all these equations are not considered for analysis as these equations are based on the parameters that are integral component of the major classification systems.

The empirical estimation of deformation of rock mass using rock mass classification systems is an interesting and open topic for many researchers and further development is in progress in this area. For the last three decades a great number of empirical equations were proposed for estimation of deformation modulus of rock mass using numerous rock mass classification systems. Bieniawski, 1978 proposed the first empirical equation to estimate deformation modulus of rock mass using in situ data and RMR classification system. The journey of development of empirical equations based on rock mass classification systems is still continued and large number of such equations based on RMR (Bieniawski, 1978), (Serafim & Pereira, 1983), (Nicholson & Bieniawski, 1990), (Mehrotra, 1992), (Kim, 1993), (Mitri, Edrissi, & Henning, 1994) (Read , Richards, & Perrin, 1999),(Mohammad , 1998), (Ramamurthy, 2001), (Ramamurthy, 2004), (Sonmez, Gokceoglu, Nefeslioglu , & Kayabasi, 2006), (Chun, Lee, Seo, & Lim, 2006)(Mohammadi & Rahmannejad, 2010) and (Shen , Karakus, & Xu, 2012) , GSI (Hoek & Brown, 1997), (Hoek, Carranza-Torres, & Corkum, 2002), (Gokceoglu, Sonmez, & Kayabasi , 2003), (Carvalho, 2004), (Sonmez, Gokceoglu, & Ulusay, 2004), (Galera, Alvarez, & Bieniawski, 2005), (Hoek & Diederich, 2006), (Beiki, Bashari, & Majdi, 2010) and (Ghamgosar, Fahimifar, & Rasouli, 2010) and Q system (Barton, 1983) (Grimstad & Barton, 1993), (Diederichs & Kaiser, 1999), (Ramamurthy, 2001), (Barton , 2002) and (Ramamurthy, 2004) have been developed so far while attempt have been made to include RMI (Palmstrøm & Singh, 2001) in the list. Majority of the equations are best fit on data for which the equations were developed. The behavior of rock mass varies and it is almost impossible to obtained such an equation that can be used globally however, it is possible to obtain more reliable general equation by including more/ new data and modify the relationship based on apparent trend of the data.

Table 1: Equation based on intact rock properties, RQD and Weathering Degree

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Eq. No.	Equation	Reference
3	$E_{rm} = 0.135\left(\frac{Ei(1+RQD/100)}{WD}\right)^{1.811}$	Kayabasi et al., 2003
4	$E_{rm} = 0.001\left(\frac{(Ei/\sigma c)(1+RQD/100)}{WD}\right)^{1.5528}$	Gokceoglu et al, 2003
5	$E_{rm} = Ei10^{0.0186RQD-1.91}$	Zhang and Einstein, 2004

2 Evaluation of the existing Empirical Equations

In this research equations Based RMR, GSI and Q classification systems are evaluated as given in Table 2. These existing equations are grouped according to the classification system on which they based and divided into groups on basis of independent variables incorporated in the relationship. In this study the empirical equations based on RMR, GSI and Q systems are grouped into ten different groups subject to the input parameters used and each group is analyzed separately. Each individual equation in the groups is reviewed in detail to check their reliability, application and limitations.

2.1 Generation of data points and Optimization of parameters

An artificial data is generated for each group separately from valid data points of existing equations instead of using the actual data of the individual equation. For selected groups where generalization is possible and required,20 data points for each equation are generated. The equations are trimmed for their valid range and a scatter data is generated for each group. Keeping in view the trend of data points in the scatter plot of individual group, Gaussian and Sigmoidal type mathematical functions are fitted to obtain generalized equation for each group that two or more than two valid equations. The new equations fitted to the data are optimized by minimizing residuals using Microsoft Excel built in Add-in “Solver”.

2.2 Prediction performance and generalization

2.2.1 RMR group I:

The first ever equation for estimation of deformation modulus of rock mass based on RMR classification system developed by Bieniawski, 1978. However, the equation is only applicable for good quality rock masses with RMR values greater than 50. Keeping in view the limitation of equation 6, Serafim and Pereira, 1983 developed a new correlation based on data from Bieniawski, 1978 and data collected by the originators of equation 7, having RMR in the range from 25 to85. The equation overestimates the deformation modulus rock mass at for lower range i.e. RMR< 10 (Mohammad , 1998) and upper range i.e. RMR > 90 (Hoek & Diederich, Empirical estimation of rock mass modulus, 2006). Mehrotra, 1992 presented a new best fit RMR based equation similar to that of Serafim and Pereira equation on his own data. Kim, 1993 proposed an exponential equation however the equation extremely overestimates deformation modulus of rock mass for RMR >85. An attempt has been made by Mohammad, 1998 to reduce the value of estimated deformation modulus equal to zero for RMR =0 by subtracting a factor of 0.562 GPa from the original Serafim, et al., 1983 equation but equation overestimate the modulus for upper range of RMR. Read , et al., 1999 developed a third power equation by keeping E_{rm}/E_i =1 for RMR =100. Chun et al, 2006 obtained an exponential relationship between RMR and deformation modulus for weak rock masses. Based on in-situ data from different parts of Iran, a polynomial equation was developed by Mohammadi and Rahmannejad, 2010 but due to poor asymptote the equation over estimates the deformation modulus RMR < 25 and overestimate for RMR approaching to 100. Shen , et al., 2012 fitted a Gaussian function on data obtained from trend of data Bieniawski, 1978, Serafim and Pereira, 1983 and (Stephens & Banks, 1989). All the equations of the group were plotted as shown in Figure 1 to highlight the performance of each equation The group comprises of 9 equations however equation 7, 8, 9, 11 and 13 are trimmed upto invalid range of RMR for which the equations were seems to overestimate the deformation modulus of rock mass. Keeping in view the trend of the data two different type of mathematical functions i.e. Gaussian and Sigmoidal are fitted to the data points as shown in Figure 2. The parameters of the models using these types functions optimized by minimizing the sum of error square i.e. residual using Solver of Microsoft Excel. The optimized general equations are given by equation (38) and (39) with residual of 3638.57 and 3612.4 respectively. It is observed that fitting of sigmoidal function bitterly described the correlation between RMR and deformation modulus of rock mass.

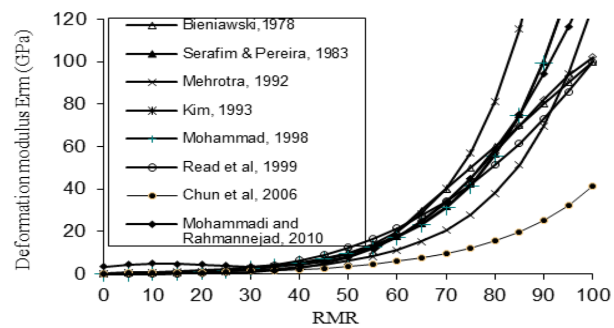

Figure 1: Comparative Plot of empirical Equations belong to RMR Group – I.

$$E_{rm(g)} = 113\, e^{-\left(\frac{RMR-113}{39}\right)^2} \quad (38)$$

$$E_{rm(s)} = \left(\frac{144}{1+e^{((86-RMR)/14)}}\right) \quad (39)$$

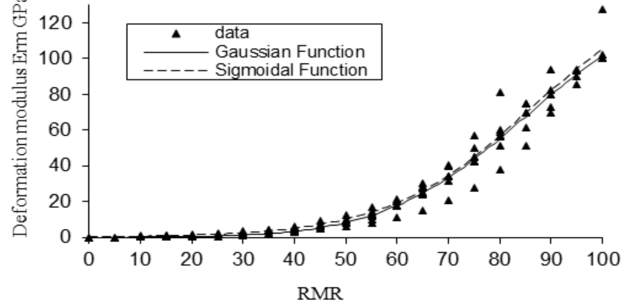

Figure 2: Fitting of equation (38) and (39) on data generated from equations of Group – I

2.2.2 RMR Group – II:

Similar to Group – I, the evaluation and refinement of equations in Group –II is continued by working on either new data or trying different mathematical functions on old data. Nicholson and Bieniawski, 1990 for the first time adopted the concept of including intact rock young modulus in relating deformation modulus of rock mass with empirical rock mass classification system RMR. Soon the idea is duplicated by Mitri et al, 1994, Ramamurthy, 2001, Ramamurthy, 2004, Galera et al, 2005, Sonmez et al, 2006 and recently by Shen et al, 2012. The idea to extend the expression of deformation modulus in this group was initially taken through correlations with observed deformations from case history studies and predicted deformations from finite element analyses. Findings of Sonmez et al, 2006 that deformation modulus of highly quality rock mass with weak intact rock pieces is highly controlled by intact rock properties has reinforced the idea to include the intact rock properties especially in equations while correlating deformation modulus of rock mass with rock mass classification system.

The prediction performance of the empirical equation are checked by plotting and analyzing the equations listed under Group – II, it is observed that equation proposed by Ramamurthy, 2004 overestimates the value of E_{rm}/E_i for lower range of RMR as illustrated by Figure 3. Gaussian and Sigmoidal type mathematical functions are fitted to the data points and optimized using Solver as shown in Figure 4. The optimized general equations are given by equation (40) and (41) with minimum residual of 2.434 and 2.383 respectively.

$$E_{rm(g)} = 1.12 E_i \, e^{-\left(\frac{RMR-124}{57}\right)^2} \quad (40)$$

$$E_{rm(s)} = 1.98 E_i \left(\frac{1}{1+e^{-((RMR-100)/24)}}\right) \quad (41)$$

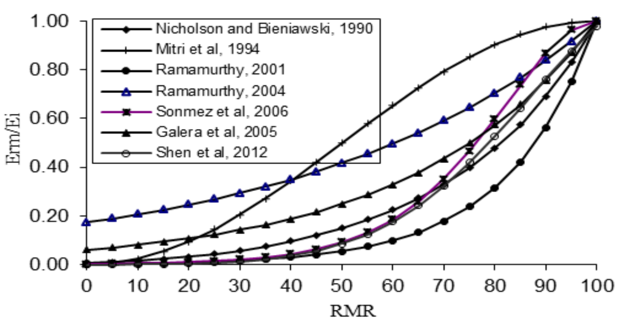

Figure 3: Comparative Plot of empirical Equations belong to RMR Group – II

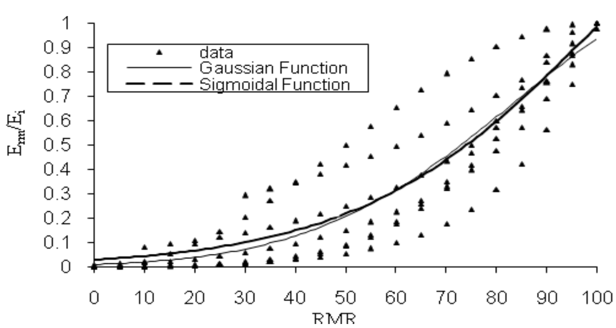

Figure 4: Fitting of equation (40) and (41) on data generated from equations of Group – II

2.2.3 GSI Group – III:

This group consists of only two published equations in which variation of deformation modulus is described as function of only GSI. The first equation was proposed by Gokceoglu, 2003 followed by Ghamgosar et al, 2010 using exponential functions however, the later extremely overestimate the deformation modulus of rock mass for GSI > 80. No further analysis is carried out due the reason that a) the Ghamgosar et al, 2010 equation is an extension of the Gokceoglu, 2003 by adding new data b) GSI only describe the quality of rock mass in terms of block size and properties of discontinuities and missing some important intact rock properties that may also responsible for controlling the deformation modulus.

2.2.4 GSI Group – IV:

The need for including an intact rock property as other controlling parameter in correlating deformation modulus of rock mass with GSI soon felt by the developer of the classification system while replacing RMR with GSI. The first equation of this group was proposed Hoek & Brown 1997 for rock masses having uniaxial compressive strength < 100 MPa. Beiki et al, 2010 developed and model based on genetic programming approach for estimation of rock mass using trigonometric function but model results negative value at both the extremes of GSI. Equations belong to this group are not used for further analysis as a) although the former equation is limited to weak rocks, it extremely overestimates the deformation modulus of rock mass due its poor asymptote and modified versions of the equation have been developed b) the second equation is based limited data with GSI range from 26 to 82 majority occurring in range of 45 to 65 and gives negative deformation modulus values when extrapolated to extremes.

2.2.5 GSI Group – V:

The basic controlling parameters of deformation modulus of rock mass in the equations of this group are assumed as σ_{ci}, Damage Factor D and GSI. Only a single published equation is available in this group is proposed by Hoek et al., 2002 keeping in view that in weak rock masses the besides the other two parameters the deformation modulus of rock mass is controlled by the intact rock strength. However, the proposed equation is limited for weak rock mass and overestimates the rock mass at upper range of GSI.

Further analysis of this group is not carried out due to a) there only a single equation in this group and no other data is available for comparison and b) the equation is limited to rock mass having σ_{ci} <100 MPa.

2.2.6 GSI Group – VI:

Keeping in view the fact the deformation modulus of rock is not controlled by intact rock properties and rock mass quality but also influenced by method of excavation. For rock masses Hoek et al, 2002 founded that the deformation modulus is greatly influenced by the damage occurred due blasting and stress redistribution for excavation and established an equation for estimation of deformation modulus of rock mass from GSI and incorporate factor D. The equation is however is limited to rock mass with intact rock uniaxial compressive σ_{ci} > 100 MPa. Based on in-situ data from China and Taiwan, Hoek and Diederichs, 2006 extended the previous equation to all rock mass assumed to be homogenous and isotropic and established new equation based on GSI, and D. Similar to many other equations this equation has good prediction performance.

The prediction performance of the empirical equation are checked by plotting and analyzing the equations listed under Group – VI, it is observed that equation proposed by Hoek et al, 2002 overestimates the value of E_{rm} for upper range of GSI as illustrated by Figure 5 . Gaussian and Sigmoidal mathematical functions are fitted to the upper and lower bound of the generated data and optimized using Solver as shown in Figure 6 and 7. Four boundary equations (equation 42 to 45), two for each function type are produced with minimum residual of 204.53 and 28.53 for upper bound and 2.19 and 0.0 respectively. Finally combining the upper and lower bound equations of each function type, two general equations proposed for all values of D i.e. from 0 to 1. The optimized general equations are given by equation (46) and (47) respectively.

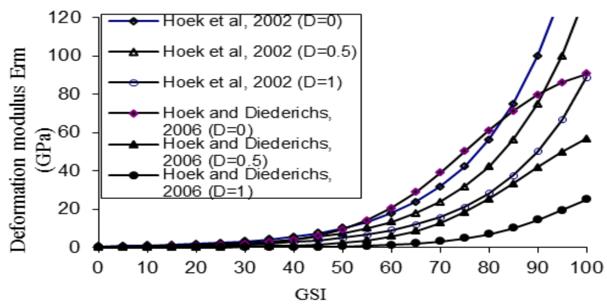

Figure 5: Comparative Plot of empirical Equations belong to RMR Group – VI

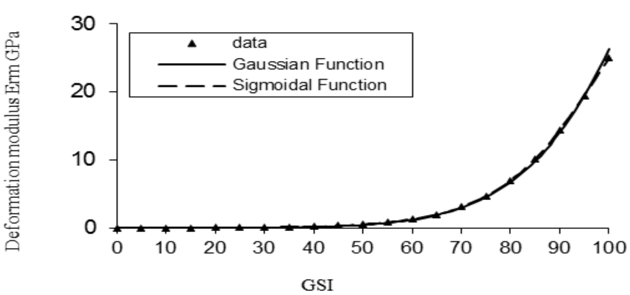

Figure 6: Fitting of Gaussian and Sigmoidal functions to lower bound data generated from equations of Group – VI

$$E_{rm(g)} = 120\, e^{-\left(\frac{GSI-120}{46}\right)^2} \quad \text{For D = 0} \quad (42)$$

$$E_{rm(g)} = 162\, e^{-\left(\frac{GSI-162}{46}\right)^2} \quad \text{For D = 1} \quad (43)$$

$$E_{rm(s)} = \left(\frac{100}{1+e^{((75-GSI)/11)}}\right) \quad \text{For D = 0} \quad (44)$$

$$E_{rm(s)} = \left(\frac{50}{1+e^{((100-GSI)/11)}}\right) \quad \text{For D = 1} \quad (45)$$

$$E_{rm(g)} = 120\,(1-42D)\, e^{-\left(\frac{GSI-120+42D}{46}\right)^2} \quad (46)$$

$$E_{rm(s)} = \left(\frac{100\,(1-D/2)}{1+e^{((75+25D-GSI)/11)}}\right) \quad (47)$$

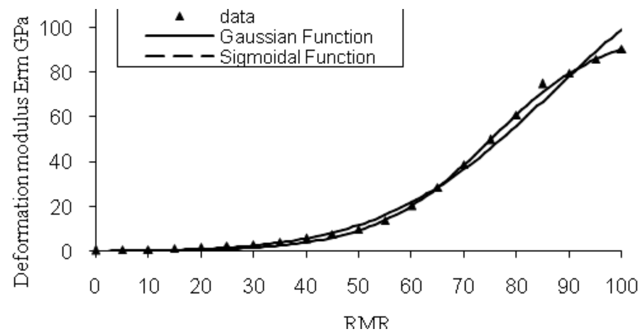

Figure 7: Fitting of Gaussian and Sigmoidal functions to upper bound data generated from equations of Group – VI

2.2.7 GSI Group – VII:

The equations in this group are based on *GSI*, E_i and *D*. Carvalho, 2004 for the first time proposed a generalized equation estimation of E_{rm} incorporating these three controlling parameters. Based on the assumption that when GSI = 100, the ratio of the modulus ratios of rock mass and intact rock is equal to unity, Sonmez et al, 2004 proposed a new prediction model for estimation E_{rm}. However in both the equations the damage factors is not incorporated properly and estimate the same value of deformation modulus for all value of D i.e. 0 to 1when GSI = 100.

Based on in-situ data from China and Taiwan, Hoek and Diederichs, 2006 extended his own equation and incorporated E_i in equation earlier presented by the authors in the same paper. This equation is assumed to be more authentic as a) based on larger in-situ tests database b) variety of rock masses are incorporated and c) The mathematical function selected truly represents the trend of the scatter data. However the use of equation can lead the user to wrong end due to a) the intact rock samples for E_i are not always taken from behind the in situ test site and are not always true representative of the rock mass and b) although guidelines for selection of D are available (Hoek et al, 2002) but it is difficult for inexperienced engineer to obtain values for damage between the given values in the guideline.

The prediction performances of the empirical equations are checked by plotting and analyzing the equations listed under Group – VII are plotted as illustrated by Figure 8. Artificial data points are generated to determine best fit generalized equation. Gaussian and Sigmoidal mathematical functions are fitted to the upper and lower bound of the generated data and optimized using Solver as illustrated by Figure 9 and 10 respectively. Four boundary equations (equation 48 to 51), two for each function type are produced with minimum residual of 0.03 and 0.02 for upper bound and 0.044 and 0.013 for lower bound respectively are obtained. Finally combining the upper and lower bound equations of each function type, two general equations proposed for all values of D i.e. from 0 to 1. The optimized general equations are given by equation 52 and 53 respectively.

$$E_{rm(g)} = E_i\, e^{-\left(\frac{GSI-100}{54}\right)^2} \quad \text{For D = 0} \quad (48)$$

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$$E_{rm(g)} = 1.52 E_i e^{-\left(\frac{GSI-152}{54}\right)^2} \quad \text{For D = 1} \qquad (49)$$

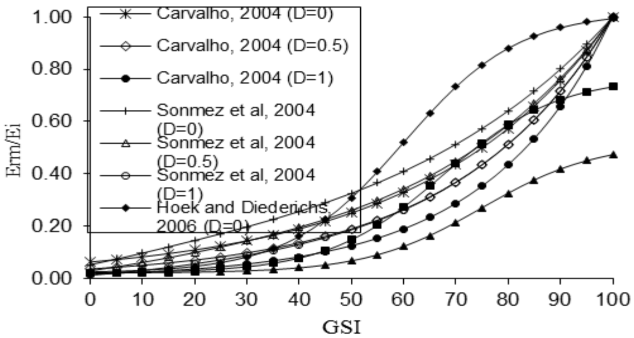

Figure 8: Comparative Plot of empirical Equations belong to RMR Group – VII

$$E_{rm(s)} = 1.15 E_i \left(\frac{1}{1+e^{((64-GSI)/19)}}\right) \quad \text{For D = 0} \qquad (50)$$

$$E_{rm(s)} = 0.6 E_i \left(\frac{0.5}{1+e^{((80-GSI)/19)}}\right) \quad \text{For D = 1} \qquad (51)$$

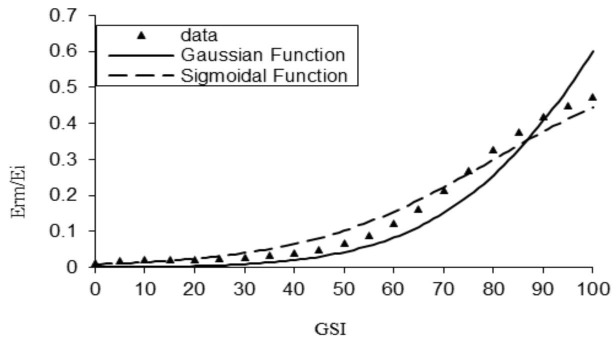

Figure 9: Fitting of Gaussian and Sigmoidal functions to lower bound data generated from equations of Group – VII

$$E_{rm(g)} = (1 + 0.52D) E_i e^{-\left(\frac{GSI-(100+52D)}{54}\right)^2} \qquad (52)$$

$$E_{rm(s)} = 1.20 E_i \left(\frac{(1-D/2)}{1+e^{((64+16D-GSI)/19)}}\right) \qquad (53)$$

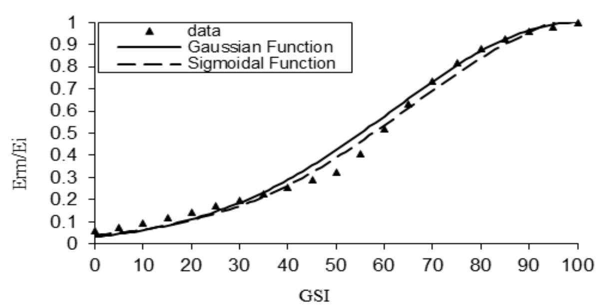

Figure 10: Fitting of Gaussian and Sigmoidal functions to upper bound data generated from equations of Group – VII

2.2.8 Q Group – VIII:

Equations based on Q as proposed Barton, 1983 and Grimstad and Barton, 1993 are for typical rock mass and typical range of Q value and estimate negative value of E_{rm} for Q < 1. The equation proposed by Diederichs and Kaiser, 1999 is not based on actual Q system. The value of Q' parameter is obtained from RMR. The equation estimates a very high value of deformation modulus for low range of RMR and very low value for upper range of RMR. Further analysis of the equations in this group is not preceded due to a) the first and last equations poorly define the deformation modulus b) only the second equation seems to be valid but for Q > 1 and c) the originator of the first equation have proposed a modified version but more general equation by adding the possible influence of intact rock properties.

2.2.9 Q Group – IX:

Two equation belong to this group are collected from literature both developed by the same researcher. Ramamurthy, 2001 proposed an equation assuming that the deformation modulus is dependent not only on Q index of rock mass and included E_i while establishing equation for E_{rm}. From the analysis of the equation it is revealed that for Q = 1000, the reduction factor i.e. E_{rm}/E_i reach just to value of 0.75. Later on Ramamurthy, 2004 modified their own equation by adding new data using the similar mathematical function. Although for Q = 1000, this equation do not achieve the theoretical E_{rm}/E_i = 1, the equation is considered to be more realistic in this group and no further analysis for development of general equation is carried.

2.2.10 Q Group – X:

Based on practical experience and case histories, Barton, 2002 proposed a modified version equation by including σ_{ci} while correlating E_m with Q index. The equation is suitably fit to data on which it was developed. In this group only one equation is available and no further generalization is required.

3. Conclusion

Analysis of this research shows that most of the equations for empirical estimation of deformation modulus of rock mass from rock mass classification systems are based on RMR. However, due to wide acceptance of GSI, the attraction to develop new/modified equations based on the classification system has been increased. In-spite of the popularity of Q system the progress and development is negligible. The reason might be the fact that the former two classification systems have broader applicability in rock engineering while the latter is limited to tunnel engineering. Majority of the equations either overestimate or underestimate the modulus at certain end and the equations need to be truncated.

Group III, IV, V, VIII, IX and X are either limited to single equation or the validity of equations in the groups is limited and not suitable for generalization. Subject to the constraints new general equations are developed for Group I, II, VI, and VII using Gaussian and Sigmoidal type functions. Eight Simplified equations using Gaussian and Sigmoidal functions are proposed. The proposed equations in this research are best fit equations but selection must be made subject to availability of controlling parameters authentic data. It is concluded from the residuals the equations using sigmoidal function give better prediction than equations using Gaussian function for all groups i.e. Group I, II, VI and VII.

References

i. *Barton , N. (2002). Some new Q value correlations to assist in site characterization and tunnel design. International Journal of Rock Mechanics and Mining Sciences, 39, 185-216.*

ii. Barton, N. (1983). Application of Q system, index tests to estimate shear strength and deformability of rock masses. Proceedings of the Internayional Symposium on Engineering Geology and Underground Construction, I (II), pp. 51-70. Lisbon, Portugal.

iii. Beiki, M., Bashari, A., & Majdi, A. (2010). Genetic Programming approch for estimating the deformation modulus of rock mass using sensitivity Analysis by neural network. International Journal of Rock Mechanics and Mining Sciences, 47, 1091-1103.

iv. Benson, C. P. (1986). Literature review of rock mass properties for analysis of navigation structures founded on rock. Master thesis, Blackburge, Virginia.

v. Bieniawski, Z. T. (1978). Determining rock mass deformability: experience from case histories. Int J Rock Mech Min Sci Geomech Abstr, 15, 237-247.

vi. Carvalho, J. (2004). Estimation of rock mass modulus (Referenced from paper by Hoek and Diederichs 2006).

vii. Chun, B., Lee, Y., Seo, D., & Lim, B. (2006). Correlation of deformation modulus by PMT with RMR and rock mass condition. Tunnelling and Underground Space Technology, 21(3-4), 231-232.

viii. Diederichs, M. S., & Kaiser, P. K. (1999). Stability of large excavations in laminated hard rock masses: the Voussoir analogue revisited. International Journal of Rock Mechanics and Mining Sciences, 36, 97-117.

ix. Galera, J. M., Alvarez, Z., & Bieniawski, Z. T. (2005). Evaluation of the deformation modulus of rock masses: comparison between pressure meter and dilatometer tests with RMR predictions. Proc. ISP5-PRESSIO 2005, eds Gambin, Magnen, Mestat and Baguelin. LCPC publication Paris.

x. Ghamgosar, M., Fahimifar, A., & Rasouli, V. (2010). Estimation of rock mass deformation modulus from laboratory experiments in Karun dam. Proceedings of the International Symposium of the International Society for Rock Mechanics (pp. 805-808). Zhao, Laboise, Dudt & Mathier (eds): Taylor & Francis Group.

xi. Gokceoglu, C., Sonmez, H., & Kayabasi , A. (2003). Predicting the deformation moduli of rock masses. International Journal of Rock Mechanics and Mining Sciences, 40, 701-710.

xii. Grimstad, E., & Barton, N. (1993). Updating of the Q-system for NMT. In: Kompen, Opsahl, Berg, editors. Proceedings of the International Symposium on Sprayed Concrete—Modern Use of Wet Mix Sprayed Concrete for Underground Support. Fagernes. Oslo: Norwegian Concrete Association.

xiii. Hoek, E., & Brown, E. T. (1997). Practical estimates of rock mass strength. International Journal of Rock Mechanics and Mining Sciences, 34(8), 1165-1186.

xiv. Hoek, E., & Diederich, M. S. (2006). Empirical estimation of rock mass modulus. International Journal of Rock Mechanics and Mining Sciences, 43, 203-215.

xv. Hoek, E., Carranza-Torres, C., & Corkum, B. (2002). Hoek - Brown failure criterion 2002 edition. Proc. NARMS - TAC Conference, (pp. 267-273). Toronto.

xvi. Jing, L. (2003). A review of techniques, advances and outstanding issues in numerical modelling for rock mechanics and rock enginneering. International Journal of Rock Mechanics and Mining Sciences, 40, 283-353.

xvii. Kayabasi, A., Gokceoglu, C., & Ercanoglu, M. (2003). Estimating the deformation modulus of rock masses: A comparative study. International Journal of Rock Mechanics and Mining Sciences, 40, 55-63.

xviii. Kim, G. (1993). Revaluation of geomechanics classification of rock masses . Proceedings of the Korean geotechnical society of spring national conference, (pp. 33-40). Seoul.

xix. Kulhawy, F. H. (1978). Geomechanical models for rock foundation settlement. J. Geot. Eng. Div. (ASCE), 104(GT2), 211-227.

xx. Mehrotra, V. K. (1992). Estimation of engineering properties of rock mass. PhD thesis, IIT Roorkee, Roorkee, India.

xxi. Mitri, H. S., Edrissi, R., & Henning, J. (1994). Fininte element modeling of cablebolted slopes in hard rock ground mining. presented at SME annual meeting (pp. 94-116). New Mexico: Albuquerque.

xxii. Mohammad , N. (1998). Subsidence numerical modelling of weak rock masses. PhD thesis, Univeristy of Nottingam, School of Chemical Environmental and Mining Engineering, Nottingam.

xxiii. Mohammadi, H., & Rahmannejad, R. (2010). The estimation of deformation modulus using regression and artificial neural network analyis. The Arabian Journal for Science and Engineering, 15.

xxiv. Nicholson , G. A., & Bieniawski, Z. T. (1990). A nonlinear deformation modulus based on rock mass classification. Int J Min Geol Eng., 8, 181-202.

xxv. Palmstrøm, A., & Singh, R. (2001). The deformation modulus of rock masses—comparisons between in situ tests and indirect estimates. Tunnelling and Underground Space Technology, 115-131.

xxvi. Ramamurthy, T. (2001). Shear strength response of some geological materials in triaxial compression. International Journal of Rock Mechanics and Mining Sciences, 38, 683-341.

xxvii. Ramamurthy, T. (2004). A geo-engineering classification for rocks and rock masses. International Journal of Rock Mechanics and Mining Sciences, 41, 89-101.

xxviii. Read , S. A., Richards, L. R., & Perrin, N. D. (1999). Applicability of Hoek - Brown failure criterion to New Zealand greywacke rocks. Proceeding of 9th International Congress on Rock Mechanics, 2, pp. 655-660. Paris.

xxix. Serafim, L. J., & Pereira, J. P. (1983). Considerations on the geomechanical classification of Bieniawski. proceedinngs of the symposium on engineering geology and underground openings, (pp. 1133-1144). Potagal, Lisboa.

xxx. Shen , J., Karakus, M., & Xu, C. (2012). A comparative study for empirical equation in estimating deformation modulus of rock masses. Tunneling and Underground Space Technology, 32, 245-250.

xxxi. Sonmez, H., Gokceoglu, C., & Ulusay, R. (2004). Indirect determination of the modulus of deformation of rock masses based on the GSI system. International Journal of Rock Mechanics and Mining Sciences, 41, 849-857.

xxxii. Sonmez, H., Gokceoglu, C., Nefeslioglu , A. H., & Kayabasi, A. (2006). Estimation of rock mass modulus: for intact rocks with an artificial neural network and for rock masses with new empirical equation. International Journal of Rock Mechanics and Mining Sciences, 43, 224-235.

xxxiii. Stephens, R. E., & Banks, D. C. (1989). Moduli for deformation studies of the foundation and abutments of the Portugues Dam—Puerto Rico In: Rock mechanics as a guide for efficient utilization of natural resources. Proceedings of the 30th US symposium, (pp. 31-38). Morgantown. Balkema, Rotterdam.

xxxiv. Zhang, L., & Einstein , H. H. (2004). Using RQD to estimate the deformation modulus of rock masses. International Journal of Rock Mechanics and Mining Sciences, 41, 337-341

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Table2: Empirical estimation of rock mass deformation modulus using rock mass classification systems

System	Group	input	Eq. No.	Mathematical function used	Empirical equation	Reference
RMR	I	RMR	6	Linear	$E_{rm} = 2RMR - 100$	Bieniawski, 1978
			7	Inverse log	$E_{rm} = 10^{(RMR-10)/40}$	Serafim and Pereira, 1983
			8	Inverse log	$E_{rm} = 10^{(RMR-20)/38}$	Mehrotra, 1992
			9	Exponential	$E_{rm} = 300\, e^{0.07RMR}\, 10^{-3}$	Kim, 1993
			10	Inverse log	$E_{rm} = 10^{(RMR-10)40} - 0.562$	Mohammad, 1998
			11	Third power	$E_{rm} = 0.1(RMR/10)^3$	Read et al, 1999
			12	Exponential	$E_{rm} = 0.3228\, e^{0.0485RMR}$	Chun et al, 2006
			13	Polynomial	$E_{rm} = 0.0003RMR^3 - 0.0193RMR^2 + 0.315RMR + 3.4064$	Mohammadi and Rahmannejad, 2010
			14	Gaussian	$E_{rm} = 110\, e^{-\left(\frac{RMR-110}{37}\right)^2}$	Shen et al, 2012
	II	RMR & Ei	15	Complex	$E_{rm} = 0.01E_i\,(0.0028RMR^2 + 0.9e^{\frac{RMR}{22.83}}$	Nicholson and Bieniawski, 1990
			16	Trigonometric	$E_{rm} = 0.5E_i\left(1 - \cos\left(\frac{\pi RMR}{100}\right)\right)$	Mitri et al, 1994
			17	Exponential	$E_{rm} = E_i\, e^{\left[\frac{RMR-100}{17.4}\right]}$	Ramamurthy, 2001
			18	Exponential	$E_{rm} = E_i\, e^{-0.0035[5(100-RMR)]}$	Ramamurthy, 2004
			19	Exponential	$E_{rm} = E_i\, e^{\left(\frac{RMR-100}{36}\right)}$	Galera et al, 2005
			20	Power	$E_{rm} = E_i\, 10^{\left(\frac{(RMR-100)(100-RMR}{4000\exp(-\frac{RMR}{100})}\right)}$	Sonmez et al, 2006
			21	Gaussian	$E_{rm} = 1.14E_i\, e^{-\left(\frac{RMR-116}{41}\right)^2}$	Shen et al, 2012
GSI	III	GSI	22	Exponential	$E_{rm} = 0.145\, e^{0.064GSI}$	Gokceoglu, 2003
			23	Exponential	$E_{rm} = 0.0912\, e^{00.866GSI}$	Ghamgosar et al, 2010
	IV	GSI and σc	24	Power	$E_{rm} = \sqrt{\frac{\sigma_c}{100}}\left(10^{\left(\frac{GSI-10}{40}\right)}\right)$	Hoek and Brown, 1997
			25	Trigonometric	$E_{rm} = \tan\left(\sqrt{1.56 + (\ln(GSI))^2}\right)\sqrt[3]{\sigma_c}$	Beiki et al, 2010
	V	GSI, σc and D	26	Power	$E_{rm} = \left(1 - \frac{D}{2}\right)\sqrt{\frac{\sigma_c}{100}} \times 10^{\left(\frac{GSI-10}{40}\right)}$ For $\sigma_c \leq 100\ MPa$	Hoek et al, 2002

	VI	GSI and D	27	Power	$E_{rm} = \left(1-\frac{D}{2}\right)10^{\left(\frac{GSI-10}{40}\right)}$ For $\sigma_c > 100\ MPa$	Hoek et al, 2002
			28	Sigmoidal	$E_{rm} = 10^5\left(\frac{1-D/2}{1+e^{(75+25D-GSI)/11}}\right)$	Hoek and Diederichs, 2006
	VII	GSI, Ei and D	29	-	$E_{rm} = E_i(s)^{\frac{3}{4}}$ Where $s = e^{\left(\frac{GSI-100}{9-3D}\right)}$	Carvalho, 2004
			30	-	$E_{rm} = E_i(s^a)^{0.4}$ Where $s = e^{\left(\frac{GSI-100}{9-3D}\right)}$ and $a = 0.5 + \frac{1}{6}\left(e^{GSI/15} - e^{20/3}\right)$	Sonmez et al, 2004
			31	Sigmoidal	$E_{rm} = E_i\left(0.02 + \frac{1-D/2}{1+e^{(60+15D-GSI)/11}}\right)$	Hoek and Diederichs, 2006
Q	VIII	Q	32	Logarithmic	$E_{rm} = 10 log Q$	Barton, 1983
			33	Logarithmic	$E_{rm} = 25 log Q$	Grimstad and Barton, 1993
			34	-	$E_{rm} = 7(\pm 3)\sqrt{Q'}$ where $Q' = 10(RMR - \frac{44}{21})$	Diederichs and Kaiser, 1999
	XI	Q and Ei	35	Exponential	$E_{rm} = E_i e^{(0.8625 log Q - 2.875)}$	Ramamurthy, 2001
			36	Exponential	$E_{rm} = E_i e^{-0.0035[250(1-0.3 log Q)]}$	Ramamurthy, 2004
	X	Q and σc	37	Third root	$E_{rm} = 10\left(\frac{Q\sigma_c}{100}\right)^{\frac{1}{3}}$	Barton , 2002