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Determining Stress Intensity Factor for Cracked Brazilian Disc Using Extended Finite Element Method

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Abstract— ***Cracked Brazilian Disc specimen is widely used to determine mode I/II and mixed mode fracture toughness of rock. In this study, stress intensity factor on crack tip in this specimen is calculated for various geometrical conditions of cracks using extended-finite element method. In this method, the crack is modeled independent of the mesh. The results show that the dimensionless stress intensity factors for pure Mode I and II increase with increasing crack length. The closed form solutions are introduced to determine the N_I and N_{II} for different geometry of crack.***

Keywords— Cracked Brazilian Disc (CBD), Stress Intensity Factor (SIF), Extended Finite Element Method (X-FEM), Mixed Mode

I. Introduction

Discontinuities always exist in a rock medium. Most of failures in rock occur due to the concentration of stress on cracks tip and propagation of them. Two principal subjects in rock fracture are toughness and mode of the fracture. Fracture toughness is the critical value of stress intensity factor on the crack tip. When the stress intensity factor exceeds this value, the crack develops and direction of crack propagation also depends on loading condition and the crack geometry. Generally in a rock medium, crack occurs in pure mode I, pure mode II or mixed mode I and II loading. In recent years, several laboratory specimens have been introduced to study fracturing in rocks. Cracked Brazilian disc (CBD) specimen due to its simple geometry, easy preparation and straight testing and loading condition, is widely being used [1-5]. Also, this specimen can be used in determination of mode I, II and mixed mode fracture toughness. Determination of fracture toughness in this specimen requires calculating stress intensity factor on the crack tip; one way for this calculation is using Finite Element Method (FEM). In this method, the discontinuity must be located on the boundary of elements and also needs implementing special mesh generation on the crack tip so that it can calculate the stress intensity factor. These problems have led to development of a new method named eXtended-Finite Element Method (X-FEM). In this method, discontinuous enrichment functions are added to the finite element approximation to account for the presence of the crack. Hence, the discontinuity is independent of the mesh, and unlike FEM, there is no need to re-mesh the domain in crack growth process. In XFEM, an initial discretization is generated and then different crack geometries are inserted into it. The extended finite element method was first proposed by [6]. They provided a method based on the finite element method to model crack growth by minimizing remeshing. Reference [7] improved this method in which the entire domain of crack is modeled completely independent of the mesh. They called this eXtended-Finite Element Method. However, the most significant and effective step in developing the extended finite element method has been achieved by [8].

Considering the importance of crack tip SIF calculation and X-FEM efficiency in problems involving crack, this study is aimed to calculate SIF on crack tip in CBD specimen for various geometrical conditions of crack (i.e. different crack length and different angels with respect to the diametrical loading) using X-FEM. An object oriented code called MEX-FEM, based on X-FEM, has been developed to simulate the crack in rock. In this code, the SIF in crack tip is calculated through the interaction integral.

The outline of this paper is as follows. Description of the CBD model is presented in Section 2. Section 3 is devoted to briefly define the X-FEM approximations for problems in linear elastic fracture mechanics. Determination of SIF in CBD specimens is presented in Section 4. Section 5 presents the conclusion of this paper.

II. Cracked Brazilian Disc specimen

CBD specimen was initially used by [9]. The geometry of the CBD specimen is shown in Figure 1. Reference [10] formulated SIF for this model geometry. This method allows testing under mode I, mode II and mixed mode I-II loading conditions by using the same specimen arrangement and the same experimental setup.

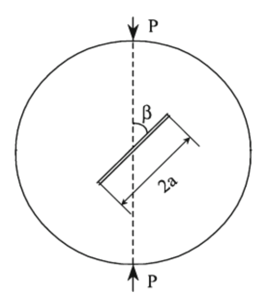


Figure 1. Geometry of the CBD specimen

The expression below is used in fracture toughness computation of CBD:

$$K_I = \frac{P\sqrt{a}}{\sqrt{\pi}RB} N_I \qquad (1)$$

$$K_{II} = \frac{P\sqrt{a}}{\sqrt{\pi}RB} N_{II} \qquad (2)$$

Where K_I is mode I stress intensity factor, K_{II} is mode II stress intensity factor, R is the radius of Brazilian disk, B is thickness of the disk, P is compressive load at failure, a is half crack length, and N_I and N_{II} are dimensionless stress intensity factors which depend on the ratio of half crack to radius (a/R) and crack orientation angles with respect to the diametrical load [4]. Reference [10] proposed the following equations for relatively small crack length (a/R≤ 0.3):

$$N_I = 1 - 4\sin^2\beta + 4\sin^2\beta(1-\cos^2\beta)(\frac{a}{R})^2 \quad (3)$$

$$N_{II} = [2 + (8\cos^2\beta - 5)(\frac{a}{R})^2]\sin 2\beta \quad (4)$$

III. Extended Finite Element Method

This method is based on finite element and partition of unity methods. The idea is to enrich the usual finite element spaces with additional degrees of freedom to account for the presence of the crack [11]. Since the mesh does not need to conform to the problem geometry, there is no need for remeshing during crack propagation. Existence of the crack causes two different enrichment types in the problem: crack interior and crack tip enrichments. The nodes whose nodal shape function support intersects the interior of the crack are enriched by a step function, and the nodes of the element which contains the crack tip are enriched by the two-dimensional linear elastic asymptotic near-tip fields (as seen in Figure 2) [12].

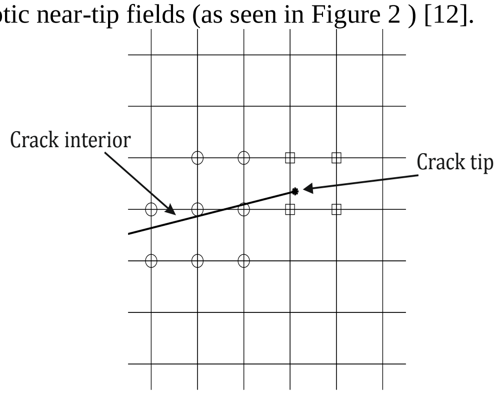


Figure.2. Illustration of a crack on mesh. The circled nodes are enriched by the Heaviside function and the squared nodes are enriched by the near-tip functions

The interior of a crack is modeled by the generalized Heaviside enrichment function H, where H takes on the value +1 above the crack and -1 below the crack:

$$H(x) = \begin{cases} +1 & for\ x > 0 \\ -1 & for\ x < 0 \end{cases} \quad (5)$$

In order to model the crack-tip and also to improve the representation of crack-tip fields, crack-tip enrichment functions are used. For an isotropic material, the crack-tip enrichment functions are given as [13]:

$$\{F_j(r,\theta)\}_{j=1,2,3,4} = \left\{\sqrt{r}\sin\frac{\theta}{2}, \sqrt{r}\cos\frac{\theta}{2}, \sqrt{r}\sin\frac{\theta}{2}\sin\theta, \sqrt{r}\cos\frac{\theta}{2}\sin\theta\right\} \quad (6)$$

Among these functions, only the first one is discontinuous indicating that discontinuity of the function is along the two faces of crack. The extended finite element approximation for the displacement field is:

$$u^h(x) = \sum_{i\in I} N_i(x)u_i + \sum_{j\in J} N_j(x)H_j(x)a_j + \sum_{k\in K} N_k(x)\sum_{l=1}^{4} F_l(x)b_k^l \quad (7)$$

where N(x) are the shape functions, ui are the nodal displacements (standard degrees of freedom), aj are vectors of additional degrees of nodal freedom associated with the Heaviside function, and b^l_k are vectors of additional degrees of nodal associated with the elastic asymptotic crack-tip functions. In the above equation, I is the set of all nodes in the mesh, J is the set of nodes enriched with discontinuous enrichment and K is the set of nodes enriched with asymptotic enrichment. Stress intensity factors are computed using domain forms of the interaction integrals:

$$I^{(1,2)} = \int_A [\sigma_{ij}^{(1)} \frac{\partial u_i^{(2)}}{\partial x_1} + \sigma_{ij}^{(2)} \frac{\partial u_i^{(1)}}{\partial x_1} - W^{(1,2)}\delta_{1j}] \frac{\partial q}{\partial x_j} dA \quad (8)$$

This integral is calculated over elements intersected by a circle of radius r_d, centered at the crack tip (i.e. the shaded elements in Figure 3). The relationship between stress intensity factors and interaction integral is:

$$I^{(1,2)} = \frac{2}{E'}(K_I^{(1)}K_I^{(2)} + K_{II}^{(1)}K_{II}^{(2)}) \quad (9)$$

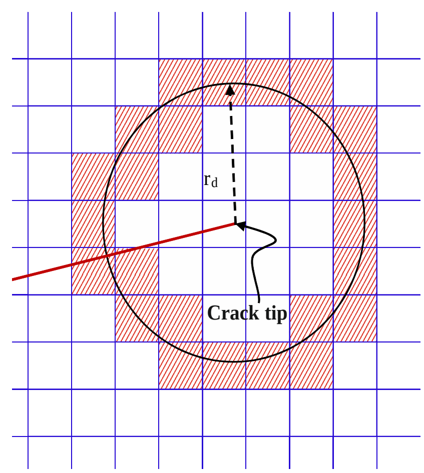


Figure 3. Illustration of elements for calculation of the interaction integral (shaded elements)

IV. Stress Intensity Factor in CBD specimen

As mentioned earlier, in XFEM, an initial discretization is generated and then different crack geometries can be considered into it. When the crack angle from loading axis is zero, the pure mode I occurs. Variations of dimensionless stress intensity factor for pure Mode I (N_I) with crack length ratio (a/R) is shown in Figure 4. N_I value of the pure mode I increases with increasing the crack length. Changing the crack angle produces different combinations of the modes I and II. Figure 5 shows the results obtained from this study for a/R = 0.1 and the results given by [10]. As seen in Figure 5, the values obtained from XFEM are in good agreement with the results by [10].

Variations of N_I and N_{II} with crack angle for different crack lengths ratio are shown in Figure 6. With increasing crack angle, N_I value decreases and N_{II} value increases to a maximum value and then decreases.

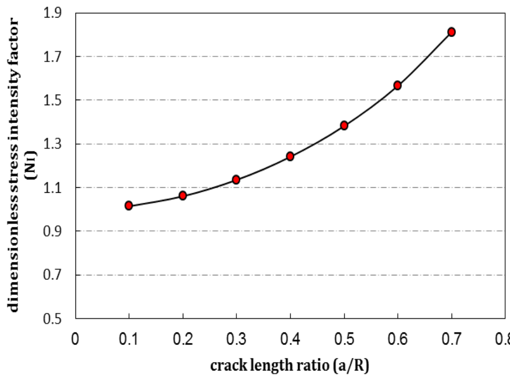


Figure 4. Variation of N_I with the crack length ratio

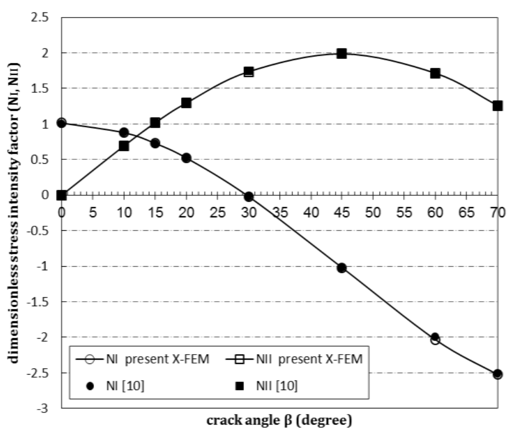
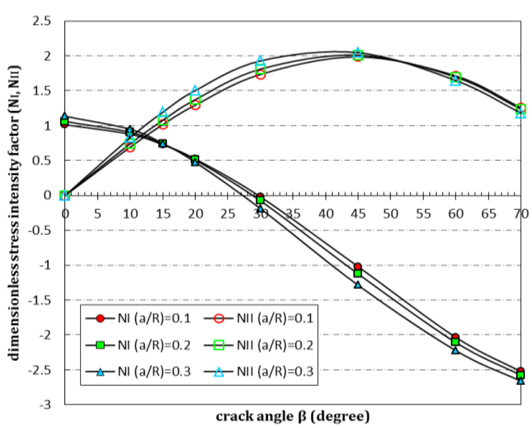


Figure 5. Variations of N_I and N_{II} with the crack angle β

(a)



(b)

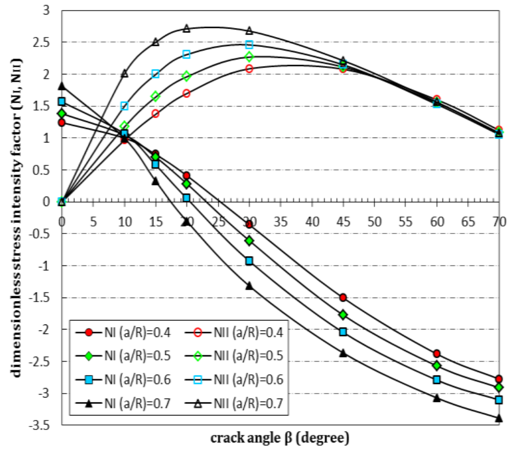


Figure 6. Variations of N_I and N_{II} with the crack angle β a) (a/R)=0.1-0.3 b) (a/R)=0.4-0.7

Theoretically, pure mode-II occurs when N_I is zero for a mixed-mode loading condition. For different crack lengths, the pure mode II occurs at different angles. These angles can be obtained from Figure 6. The angles of pure mode II for various crack lengths have been shown in Figure 7. Note that the angle of pure mode II decreases with increasing crack length. Figure 8 shows the variation of N_{II} value of pure mode II with crack length ratio. N_{II} value of pure mode II increases with increasing crack length.

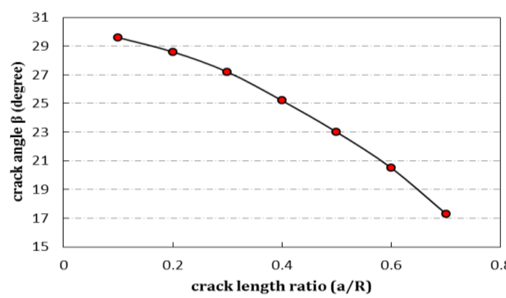


Figure 7. Variation of the crack angle of pure mode II with (a/R)

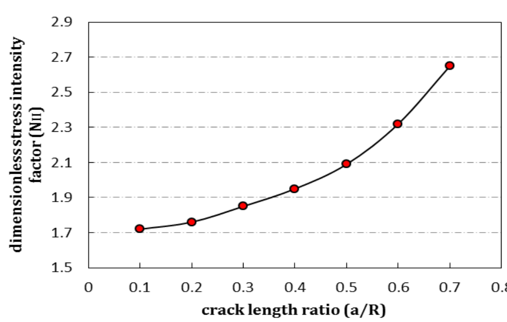


Figure 8. Variation of N_{II} with the crack length ratio

These data were subjected to regression analysis by fitting to polynomial functions. The value of N_I and N_{II} for any crack length ratio can be calculated by fourth and fifth order polynomial functions in terms of crack angle.

$$N_I = C_1 \sin^4 \beta + C_2 \sin^3 \beta + C_3 \sin^2 \beta + C_4 \sin \beta + C_5 \quad (10)$$

$$N_{II} = D_1 \sin^5 \beta + D_2 \sin^4 \beta + D_3 \sin^3 \beta + D_4 \sin^2 \beta + D_5 \sin \beta + D_6 \quad (11)$$

Coefficients Ci and Di of the polynomial fits are tabulated in Tables 1 and 2, respectively.

Table 1. Coefficients Ci used for N_I estimation based on polynomial fits

a/R	C_1	C_2	C_3	C_4	C_5	R^2
0.1	1.0851	-1.817	-3.0185	-0.2333	1.0153	1
0.2	1.348	-1.4497	-3.7182	-0.2192	1.0609	1
0.3	0.701	1.0437	-5.798	-0.095	1.1352	1
0.4	-0.9769	5.7156	-9.108	0.0475	1.2406	1
0.5	-4.366	13.589	-13.906	0.121	1.3815	1
0.6	-10.276	24.777	-19.147	-0.3427	1.5688	1
0.7	-15.307	32.336	-20.288	-2.3515	1.8194	0.9998

Table 2.Coefficients Di used for N_{II} estimation based on polynomial fits

a/R	D_1	D_2	D_3	D_4	D_5	D_6	R^2
0.1	-15.588	29.998	-23.261	6.0358	3.474	0.0005	0.9999
0.2	-14.697	29.239	-23.742	6.1	3.7498	0.0007	0.9999
0.3	-12.083	24.903	- 21.5	4.8689	4.4263	0.0006	0.9999
0.4	-13.451	30.088	-26.484	5.1015	5.3078	0.0005	1
0.5	-16.048	37.355	-31.103	3.3797	6.9563	-0.0001	1
0.6	-21.444	47.49	-32.71	-2.7302	9.8833	-0.0004	0.9999
0.7	-16.047	23.608	4.5588	-27.841	16.195	-0.0005	1

V. Conclusion

In this study, the extended finite element method is used to evaluate stress intensity factor for various geometry of crack in CBD specimen. This method is based on the FEM and employs the enrichment functions for crack modeling independent of element. Different crack geometries can be easily inserted in a similar discretization. These results coincide with previous results in the literature. The results show that with increasing crack length, the dimensionless stress intensity factors for pure

Mode I and II increase, while the angle of pure mode II decreases. For mixed mode loading, value of N_I decreases with increasing crack angle whereas N_{II} value increases to a maximum and then decreases. The results of this study also demonstrate the potential of the X-FEM to simulate the crack in rock materials.

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