

A Novel Dynamic Model for a Crane System with a Flexible Cable and Large Swing Angle

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Abstract: In this paper a novel dynamical model will be developed for an overhead crane with transverse vibrations of flexible cable while considering large angle of the cable/payload swing. The major problem addressed is expressing the dynamic interaction between the cables and other elements in the crane system which demands investigation into boundary induced cable vibrations and large swing angle. The developed model is achieved as an ODE model and it is appropriate enough to design suitable control systems. Thus, to perform the modeling, Rayleigh-Ritz discretization method is used to achieve an ODE model for transverse deflection of the cable having finite generalized degrees of freedom. Using the Euler-Lagrange formulation, the dynamical model of the crane system with a flexible cable is obtained. The simulations are performed based on OpenSees model and the developed model to validate the obtained model. To demonstrate the dynamic behavior of crane system, an illustrative example of the crane system with flexible cable moving a lightweight and heavyweight payload is investigated under various conditions.

Keywords—Overhead crane system, Flexible cable, Cable vibration, Large swing angle, Transverse vibration

I. Introduction

The crane systems are extensively employed in a variety of applications in industries such as land and onshore/offshore construction sites, transportation industry, etc. The most common operation of a crane is point-to-point moving of a suspended load horizontally, using the cables and a support mechanism. The cables possess an inherent flexibility and can only develop tension; they do not offer resistance to bending moments or compressive forces. Such natural features certainly cause deflection in transversal direction of the cable and payload swing in crane systems. The suspended load in crane systems is always subject to swings happen by unskilled operators or by disturbances typically induced by motor drive transients, wind, and collision with objects so that it can cause lengthy transportation activities; even the swings may become large and reduce the safety.

Abdel-Rahman and Nayfeh presented a detailed review of the challenges in modeling and control of the crane systems [1]. In most dynamical models, the effects of flexibility and weight of the suspended cable have been disregarded and the

cable has been considered as a mass-less rigid-link or as a rigid-link including a point-mass [2-14]. In these studies, the payload swing has been assumed as the major dynamic motion of crane systems. Although such assumptions are usual, they are not genuine for many applications. In certain cases, especially when payload is lightweight and more importantly when cable is long, the effect of flexibility has to be taken into account. In these cases, the tension force is more dependent on the cable weight and also it will be varying along the cable. Under such conditions, the tension force, especially at the end of the cable is low so that the cable weight can possibly have more dynamical effects and transverse vibrations of the cable may administrate the behavior of the crane [15]. Thus, to utilize the crane systems in a particular application, in addition to the payload swing, the cable vibration should be suppressed within a given period of time for safety issues; therefore, development of an effective suppression control system is essential. In order to achieve these objectives, a more practical and accurate model with more details including interactive dynamics of the cable and the payload is required.

A few studies have addressed the effects of cable flexibility in crane systems [16-26]. They have offered planar models for overhead cranes and assumed that the cable is perfectly flexible and inextensible. Also, they have assumed that the cable slope is small along the cable and also disregarded the swing angle; in other words, in these studies, in a crane system, the major dynamic motion of cable has been considered to be the cable vibrations. However, this is valid only for slow movements of the trolley or support mechanism and near the end of the traveling. These models might not be accurate enough when considering certain unavoidable physical and environmental conditions. The angular rotation of cable swing in most applications especially when a high speed traveling is required becomes large. This issue has been disregarded in nearly all of the previous studies on the crane system with a flexible cable [17-26]. In an infrequent study, an approximated model was established by introducing the effect of small swing angle just in the trolley dynamics in a crane system with flexible cable [16]. Here, an attempt to extract an accurate dynamical model that exactly represents the dynamic interaction effects of the payload and suspended cable and no particular assumption is made regarding the major dynamics of a crane system and both cable vibrations and payload/cable swing will be considered in dynamic modeling of the crane system. Moreover, the swing angle of the cable/payload is not being limited to small angles. Thus, a more accurate dynamical model including both the

transverse vibrations and large swing angle will be obtained. To do so, an overhead crane system with a flexible cable is considered and during the crane travel, it is assumed that the cable is inextensible with no load hoisting or lowering. The cables can be assumed as one-dimensional continuum elements which can be modeled approximately, assuming to have only a few modes of vibration [27-29]. One frequently used method for approximate modeling of continuum elements is Rayleigh-Ritz discretization. This method produces an approximate solution in the form of a finite series consisting of known shape functions or trial functions multiplied by undetermined time-dependent “generalized coordinates” [28]. Accordingly, the cable transverse deflection can be modeled by a finite number of generalized degrees of freedom and a set of ODEs using Euler-Lagrange formulation, which permits the determination of the generalized coordinates.

The purpose of this study is to develop a novel dynamic model for a crane system with a flexible cable while considering the cable transverse vibrations and large angle of the cable/payload swing. The major problem addressed is expressing the dynamic interaction between the cables and other elements in the crane system which demands investigation into boundary induced cable vibrations and large swing angle. To attain an ODE model, Rayleigh-Ritz discretization method is used for transverse deflection of the cable having finite generalized degrees of freedom. Using the Euler-Lagrange formulation, the dynamical model of the crane system with a flexible cable is obtained. To demonstrate the effectiveness of the control system, the numerical simulations are performed using commercial software. The simulations are performed based on OpenSees model and developed model to validate the obtained model. To demonstrate the dynamic behavior of crane system, an illustrative example of the crane systems with flexible cable moving a lightweight and heavyweight payload are investigated under various conditions.

This paper is organized as follows: The system description and kinematic analysis are introduced in Section 2. In Section

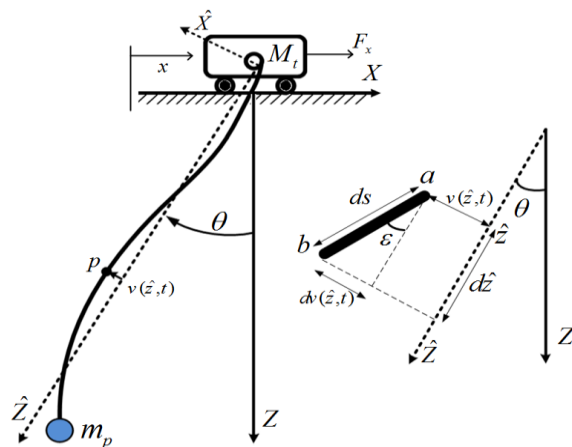


Figure 1: Coordinate frames and the schematic of the overhead crane system. An infinitesimal element ds at general point p .

3, the energy functions are introduced and the dynamic equations of motion are derived using Rayleigh-Ritz discretization method and Euler-Lagrange formulation. The several numerical simulations are performed Section 4, to illustrate and verify the behavior of a sample flexible cable crane system. Finally, in Section 5, the conclusions are drawn.

II. System Description and Kinematic Analysis

An overhead crane system is composed of a support mechanism as a trolley and a flexible cable tied to the suspended payload. The swing motion of the payload and transverse vibrations of cable can be described in a planar form using two coordinate frames i.e., XZ and $\hat{X}\hat{Z}$, see Figure 1. A general point p along the cable is defined to describe the position of a general point on the cable. The point p is located on a small element of length ds with the left end placed at point a and the right at point b . As shown, three kinds of motion can be considered, i.e. crane traveling, swing angle and transverse vibration which are described as x , θ , and $v(z, t)$, respectively. It is assumed that every point on the cable has two degrees of freedom; one is the transverse deflection $v(z, t)$ around the \hat{Z} -axis and the other is the swing angle, θ , which is not assumed to be small in this study. Let x and F_x be the trolley position and trolley driving force, respectively. The parameters M_t, m_p, ρ and g are the total trolley mass, payload mass, mass per unit length of cable and gravitational acceleration, respectively. The cable is supposed to be inextensible and the transverse deflection is assumed to be small. The payload is considered as a point-mass and the motion of the trolley on the rail is assumed to be frictionless. According to Figure 1, the position vector of the point p in the base frame XZ can be described as:

$$\begin{aligned} \vec{r}_p &= \begin{bmatrix} \cos(\vartheta) & \sin(\vartheta) \\ -\sin(\vartheta) & \cos(\vartheta) \end{bmatrix} \begin{pmatrix} v(\hat{z}, t) \\ \hat{z} \end{pmatrix} + \begin{pmatrix} x \\ 0 \end{pmatrix} \\ &= \begin{pmatrix} \cos(\vartheta)(\hat{z}, t) + \sin(\vartheta)\hat{z} + x \\ -\sin(\vartheta)(\hat{z}, t) + \cos(\vartheta)\hat{z} \end{pmatrix} \end{aligned} \quad (1)$$

where, x, \hat{z} are, respectively, the trolley position and the component of the general point p position along the \hat{Z} -axis. $v(\hat{z}, t)$ is the transverse deflection of the general point p and $R(\vartheta)$ is the rotation matrix, in which $\vartheta = \theta + \varepsilon$ is the total angular displacement of point p . As shown in Figure 1, θ and ε are defined around \hat{Z} -axis respectively are the payload swing angle and the cable slope at point p . It is assumed that the deflection of the cable is small, thus, the angle ε compared with the angle θ is small, and therefore $\vartheta \cong \theta$. The velocity of the point p can be obtained as follows:

$$\begin{aligned} \dot{\vec{r}}_p &= \dot{\theta} \begin{bmatrix} -\sin(\theta) & \cos(\theta) \\ -\cos(\theta) & -\sin(\theta) \end{bmatrix} \begin{pmatrix} v \\ \hat{z} \end{pmatrix} \\ &\quad + \begin{bmatrix} \cos(\theta) & \sin(\theta) \\ -\sin(\theta) & \cos(\theta) \end{bmatrix} \begin{pmatrix} v_t \\ \hat{z}_t \end{pmatrix} + \begin{pmatrix} \dot{x} \\ 0 \end{pmatrix} \end{aligned} \quad (2)$$

where, v_t and \hat{z}_t denote the partial derivative of v and \hat{z} with respect to time, respectively. Since it is assumed that the cable is inextensible, all points along the cable have the same axial velocity, thus, $\hat{z}_t = \dot{\hat{z}} \cong 0$, and, given these assumptions, the approximated velocity of point p is calculated as follows:

$$\dot{\vec{r}}_p = \begin{pmatrix} \dot{x} - \dot{\theta} \sin(\theta) v + \dot{\theta} \cos(\theta) \hat{z} + \cos(\theta) v_t \\ -\dot{\theta} \cos(\theta) v - \dot{\theta} \sin(\theta) \hat{z} - \sin(\theta) v_t \end{pmatrix} \quad (3)$$

To attain an ODE model describing the transverse deflection of a cable with finite degrees of freedom (modes), the Rayleigh-Ritz discretization method can be used [28]. Based on Rayleigh-Ritz discretization, the spatial function $v(z, t)$ is approximated as the finite sum of shape functions $\phi_j(z)$ multiplied by the time-dependent generalized coordinates $\mu_j(t)$ as follows:

$$v(\hat{z}, t) = \sum_{j=1}^m \mu_j(t) \phi_j(\hat{z}) = P^T(\hat{z}) \mu(t) \quad (4)$$

where, $P(\hat{z})$ and $\mu(t)$ are:

$$P(\hat{z}) = [\phi_1(\hat{z}) \quad \phi_2(\hat{z}) \quad \dots \quad \phi_m(\hat{z})]^T \quad (5)$$

$$\mu(t) = [\mu_1(t) \quad \mu_2(t) \quad \dots \quad \mu_m(t)]^T$$

Thus, the partial derivatives of $v(\hat{z}, t)$ with respect to time and \hat{z} can be calculated as:

$$v_t(\hat{z}, t) = P^T \dot{\mu}$$

$$v_{\hat{z}}(\hat{z}, t) = \frac{\partial}{\partial \hat{z}} P^T \mu, \quad v_{\hat{z}\hat{z}}(\hat{z}, t) = \frac{\partial^2}{\partial \hat{z}^2} P^T \mu \quad (6)$$

To choose the shape functions $\phi_j(z)$, the boundary conditions must be satisfied. It is assumed that the boundary conditions are $v(0, t) = v(\ell, t) = 0$ so the shape functions can be chosen as $\phi_j(\hat{z}) = \sin\left(\frac{j\pi}{\ell} \hat{z}\right)$, $j = 1, 2, 3, \dots$. To obtain the velocity of point p, the following relationship can be used.

$$\dot{r}_p = \begin{pmatrix} \dot{x} - \dot{\theta} \sin(\theta) P^T \mu + \dot{\theta} \cos(\theta) \hat{z} + \cos(\theta) P^T \dot{\mu} \\ -\dot{\theta} \cos(\theta) P^T \mu - \dot{\theta} \sin(\theta) \hat{z} - \sin(\theta) P^T \dot{\mu} \end{pmatrix} \quad (7)$$

In the following subsections, these formulations will be used to obtain the potential and kinetic energies required when extracting the dynamic equation based on Euler-Lagrange formulation.

III. Dynamic Equations of Motion

Let an infinitesimal element ds be on the point p along the cable, as shown in Figure 1, where, $ds = \sqrt{(d\hat{z})^2 + (dv)^2}$. It is assumed that the cable longitudinal elastic deformation is neglected. The cable transverse deflection is also assumed to be small, thus, $ds \cong d\hat{z}$. So, the kinetic energy of the crane system can be expressed as:

$$K = \frac{1}{2} \rho \int_0^\ell \dot{r}_p^T \dot{r}_p d\hat{z} + \frac{1}{2} M_t \dot{x}^2 + \frac{1}{2} m_p (\dot{r}_p^T \dot{r}_p)_{\hat{z}=\ell} \quad (8)$$

where, the first term denotes the kinetic energy of the cable and the other two terms are related to the trolley and the payload, respectively. The potential energy of the system is due to the payload and the cable gravity effects, and the cable transverse deflection. The potential energy due to the cable transverse deflection can be obtained as:

$$U = \int_0^\ell \rho g (\hat{z} - \hat{z} \cos(\theta) + v \sin(\theta)) d\hat{z} + \frac{1}{2} \int_0^\ell T(\hat{z}) w_{\hat{z}}^2 d\hat{z} + m_p g (\ell - \ell \cos(\theta) + v(\ell, t) \sin(\theta)) \quad (9)$$

where, the first term is the potential energy due to the cable transverse deflection, second and third terms is potential energy due to gravity effects of the cable and payload, respectively. $T(\hat{z})$ is the tension force along the cable, $w_{\hat{z}}$ is the partial derivative of $v(\hat{z}, t)$ with respect to \hat{z} , and ℓ is the

cable length. Since the cable is hung under its own weight and also weight of the payload, the tension in the cable at point p can be approximately obtained as:

$$T(\hat{z}) \cong (m_p g + \rho g (\ell - \hat{z})) \cos(\theta) \quad (10)$$

It has been assumed that both the deflection and slop of the cable around \hat{z} -axis are small, thus, $\cos(\varepsilon) \cong 1$, and $\sin(\varepsilon) \cong \varepsilon \cong \tan(\varepsilon) = v_{\hat{z}}$, so,

$$\cos(\vartheta) = \cos(\theta + \varepsilon) \cong \cos(\theta) - \tan(\varepsilon) \sin(\theta) \cong \cos(\theta) - v_{\hat{z}} \sin(\theta) \quad (11)$$

Thus, the cable tension can be obtained approximately as:

$$T(\hat{z}) \cong T_0(\hat{z}) \cos(\theta) - T_0(\hat{z}) \sin(\theta) v_{\hat{z}} \quad (12)$$

where, $T_0(\hat{z}) = (m_p + \rho \ell - \rho \hat{z}) g$. Therefore, the total potential energy can be approximately written as following where, $U_0 = \frac{1}{2} \rho g \ell^2 + m_p g \ell$ is a constant value.

$$U \cong \frac{1}{2} \int_0^\ell T_0(\hat{z}) \cos(\theta) v_{\hat{z}}^2 d\hat{z} + \int_0^\ell \rho g (\hat{z} - \hat{z} \cos(\theta)) d\hat{z} + m_p g \ell - m_p g \ell \cos(\theta) \quad (13)$$

$$= \frac{1}{2} \int_0^\ell T_0(\hat{z}) \cos(\theta) v_{\hat{z}}^2 d\hat{z} - \rho g \int_0^\ell \hat{z} \cos(\theta) d\hat{z} - m_p g \ell \cos(\theta) + U_0$$

To obtain crane's equations of motion based on the Euler-Lagrange formulation, the system Lagrangian as a function of the trolley motion, cable swing (pendulum) motion and cable (discretized) deflection will be used. This Lagrangian function is given as:

$$L = K - U$$

$$= \int_0^\ell \left(\frac{1}{2} \rho (\dot{r}_p^T \dot{r}_p) - \frac{1}{2} (m_p g + \rho g \ell) \cos(\theta) v_{\hat{z}}^2 + \left(1 + \frac{1}{2} v_{\hat{z}}^2\right) \rho g \hat{z} \cos(\theta) \right) d\hat{z} \quad (14)$$

$$+ \frac{1}{2} M_t \dot{x}^2 + \frac{1}{2} m_p (\dot{r}_p^T \dot{r}_p)_{\hat{z}=\ell} + m_p g \ell \cos(\theta) - U_0$$

Assuming that, $q = (x, \theta, \mu)^T \in R^n$ in which $\mu = (\mu_1, \mu_2, \dots, \mu_m) \in R^m$ is generalized coordinates of cable, and $n = m + 2$. To attain the dynamic equation, Euler-Lagrange equation is used as:

$$\frac{d}{dt} \left(\frac{\partial L}{\partial \dot{q}} \right) - \frac{\partial L}{\partial q} = W \cdot u \quad (15)$$

By inserting the Lagrangian function (14), $L(q, \dot{q})$, into Euler-Lagrange equation (15), the dynamic equations of the crane system will be obtained in a matrix form as:

$$M(q) \ddot{q} + C(q, \dot{q}) \dot{q} + G(q) = W u \quad (16)$$

where, $W = [1 \quad 0 \quad \dots \quad 0]^T$, $u = F_x$ is the trolley driving force and $q = (x, \theta, \mu)^T \in R^n$ in which $\mu = (\mu_1, \mu_2, \dots, \mu_m) \in R^m$ is generalized coordinates of cable, and $n = m + 2$. Matrix $M(q)$ is the total inertial matrix, $C(q, \dot{q}) \dot{q}$ represents the Coriolis and centripetal forces and $G(q) = \frac{\partial U}{\partial q}$ is the potential force due to stiffness and gravitation effects in the system. The symmetric inertia matrix, $M(q)$, can be assembled and simplified as:

$$M(q) = \begin{bmatrix} a_0 & a_1 \cos(\theta) & \cos(\theta) a_2 \\ a_1 \cos(\theta) & a_4 + \mu^T a_3 \mu & a_5 \\ \cos(\theta) a_2^T & a_5^T & a_3 \end{bmatrix} \quad (17)$$

Also, matrix $C(q, \dot{q})$ and vector $G(q)$ can be assembled as:

$$C(q, \dot{q}) = \begin{bmatrix} 0 & c_{12} & -a_2 \sin(\theta) \dot{\theta} \\ 0 & \mu^T a_3 \dot{\mu} & \dot{\theta} \mu^T a_3 \\ 0 & -\dot{\theta} a_3 \mu & 0 \end{bmatrix} \quad (18)$$

$$G(q) = \begin{pmatrix} 0 \\ g_1 \sin(\theta) + \sin(\theta) \mu^T g_2 \mu \\ \cos(\theta) g_2 \mu \end{pmatrix}$$

in which, $c_{12} = -a_1 \sin(\theta) \dot{\theta} - a_2 \mu \cos(\theta) \dot{\theta} - \sin(\theta) a_2 \dot{\mu}$ and the respective parameters are:

$$\begin{aligned} a_0 &= M_t + m_p + \rho g \ell & a_1 &= m_p \ell + \frac{1}{2} \rho \ell^2 \\ a_2 &= \rho \int_0^\ell P^T d\hat{z} & a_3 &= \rho \int_0^\ell (P^T P) d\hat{z} \\ a_4 &= m_p \ell^2 + \frac{1}{3} \rho \ell^3 & a_5 &= \rho \int_0^\ell \hat{z} P^T d\hat{z} \\ g_1 &= \left(m_p \ell + \frac{1}{2} \rho \ell^2 \right) g \\ g_2 &= \left(m_p g + \rho g \ell \right) \int_0^\ell \left(\frac{\partial}{\partial \hat{z}} P \right) \cdot \left(\frac{\partial}{\partial \hat{z}} P \right)^T \cdot d\hat{z} \\ & - \rho g \int_0^\ell \hat{z} \left(\frac{\partial}{\partial \hat{z}} P \right) \left(\frac{\partial}{\partial \hat{z}} P \right)^T \cdot d\hat{z} \end{aligned} \quad (19)$$

These nonlinear coupled ordinary differential equations describe the dynamic motions of the whole crane system with flexible cable. It is clear that q is the configuration variable vector of the system with $q_1 = x$ is the actuated variable and $q_2 = (\theta, \mu)^T$ is the un-actuated variable vector of the system. Since there are three configuration variables to be controlled with only one actuated configuration variable, the flexible cable crane system is an underactuated system. There are some general properties of inertia matrix $M(q)$, note that this matrix is symmetric, and positive definite for all q . Calculating $\dot{M} - 2C$ is a skew-symmetric matrix which has an important property.

IV. Numerical Simulation to Verify the Model

In this part, several numerical simulations are performed to illustrate and verify the behavior of a sample flexible cable crane system with a lightweight and heavyweight payload. To do so, an OpenSees computer model is used to compare and validate the developed model. Since the crane system's degrees of freedom are cable deflection, payload/cable swing angle and trolley movement, time histories of these motions will be plotted to illustrate the behavior of the sample crane system. To collect the simulation results, the obtained ODE model with four finite modes, as described in the previous section for a cable with suspended payload, is used. Also, a model with 100 dimensional finite elements which is built in OpenSees software is applied to investigate and compare the developed model. Assuming that there is no load hoisting or lowering during crane travel, two cases are simulated to verify the obtained model. First, a crane with a flexible cable and lightweight payload and, then, the same crane with a heavy payload are examined. For example, the trolley mass is 10 (kg), cable length is 5 (m) and its mass in per unit length

is 0.62 (kg/m), the mass of different payload in two cases are chosen 2 (kg) (light payload case) and 50 (kg) (heavy payload case). It is assumed that the crane system is initially at rest. To excite the cable-payload assembly, the same acceleration signal as a positive and negative pulse is imposed on the trolley (Figure2). To show traveling displacement and transverse deflection of the cable, five chosen generalized points with the same distance along the cable are considered, where traveling displacement related to the end of cable is payload motion. Assuming that the crane system is in the rest condition before moving trolley, the simulation results are plotted in Figure and Figure, to illustrate horizontal displacement of the cable and payload in line with the X-axis (horizon) for two case cranes. As seen, the dynamic behaviors of crane system based on developed model and OpenSees model are close together and the obtained model, according to the previous section with four finite degrees of freedom yields good results. To investigate the other dynamic behavior of crane system, another simulation is performed. In another way, the same crane system is excited via the same trolley force as a positive and negative pulse (Figure3). The dynamic behavior of crane system is shown in Figure4 and Figure7 to Figure9 for two cases with lightweight and heavy payloads. To determine the transverse deflection of the cable, the first four modes of vibrations are considered. The time histories of the amplitudes of these first four modes are plotted in Figure4 and Figure6. This figure clearly shows that the amplitudes are diminishing with the number of the mode, and thus, justifies using the first four modes for the approximated system in our simulations. In Figure8 and Figure9 the horizontal displacements of the trolley and the payload along X-axis is displayed. Also, Figure presents the payload swing angle. The results show that the swing angle can become large and it cannot be disregarded. To investigate the effectiveness of the payload mass, the time histories of transverse deflections of the cable is shown in Figure5 and Figure7. The results both illustrate the crane system behavior and the amount of effectiveness of payload weight on cable vibrations. As shown, when the payload is lightweight, the transvers deflection of cable and its effectiveness on payload swing become more because the tension force along the cable is not large enough, especially at the end of the cable which is allowed to have more open dynamical behavior. Obviously, when a heavy payload is tied to the end of cable, tension force on the entire length of the cable is large, so the transvers deflections of cable are small and the effectiveness of cable dynamic on payload can be ignored. Under such conditions, cable can be assumed as a rigid-link.

V. Discussion and Conclusion

This study has developed a comprehensive and convenient dynamical model for an overhead crane with a flexible cable while allowing large payload/cable swing angles. The cable was assumed to be a one-dimensional continuum element. To discretize the governing equations of this continuum, the cable transverse vibrations were approximated with finite generalized degrees of freedom based on Rayleigh-Ritz discretization method and a nonlinear model for the whole crane system was

developed using Euler–Lagrange equation. As it can be seen, the dynamic responses of the crane system based on the developed model and OpenSees model are very similar and the obtained model generates decent results. The results both illustrate the crane system behavior and the amount of effectiveness of payload weight on swing angle and cable vibrations. As shown, when the payload is lightweight, the transverse deflection of cable and its effectiveness on payload swing become more significant. Thus, considering both dynamic behavior of the swing angle and transverse vibrations are important and cannot be disregarded. But, when a heavy payload is tied to the end of cable, the transverse deflections of cable are small and the effectiveness of cable dynamic on payload can be ignored, thus, under such conditions, the major dynamic behavior of cable is swing angle and cable can be assumed as rigid-link.

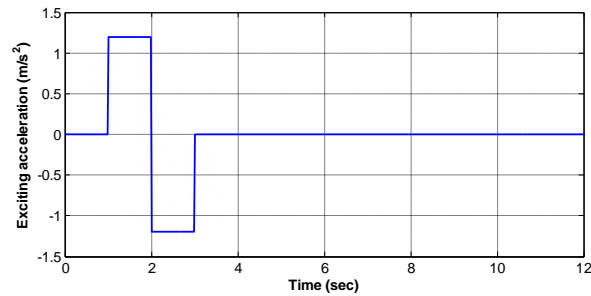


Figure2: The exciting acceleration signal on the trolley

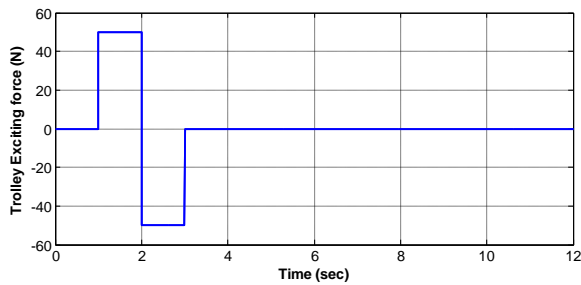


Figure3: The exciting input force signal applied on the trolley

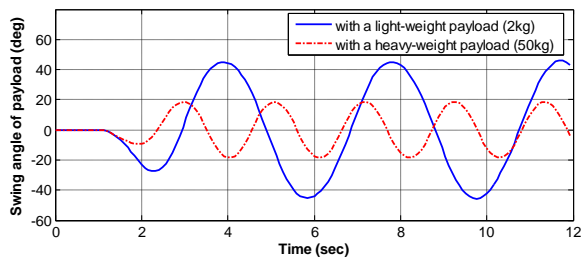


Figure4: Time history of the swing angle for the same crane system with 2kg and 50kg payloads under the exciting input force

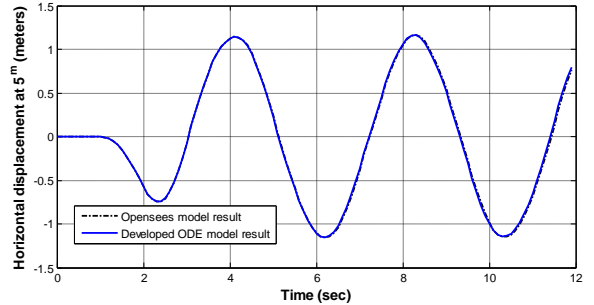
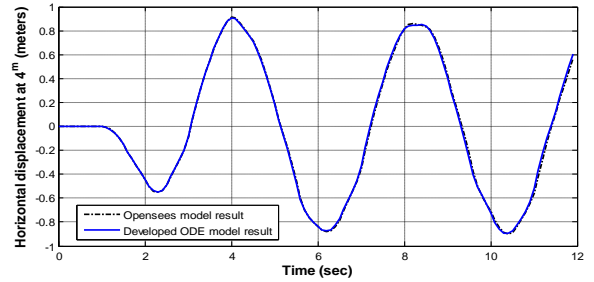
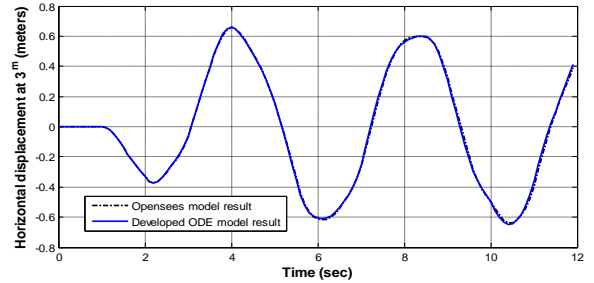
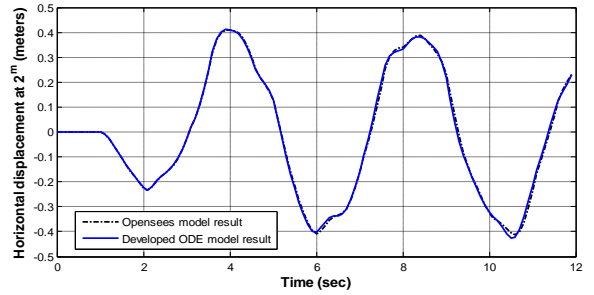
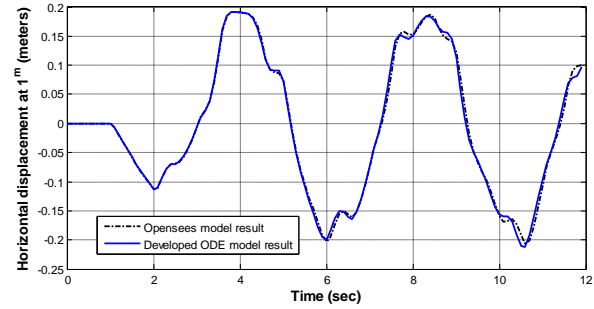


Figure5: The comparison of the simulation results using the OpenSees model and developed model. The horizontal displacements of five points, located on the cable at 1m, 2m, 3m, 4m and 5m, around vertical axis for a crane system with 2kg payload

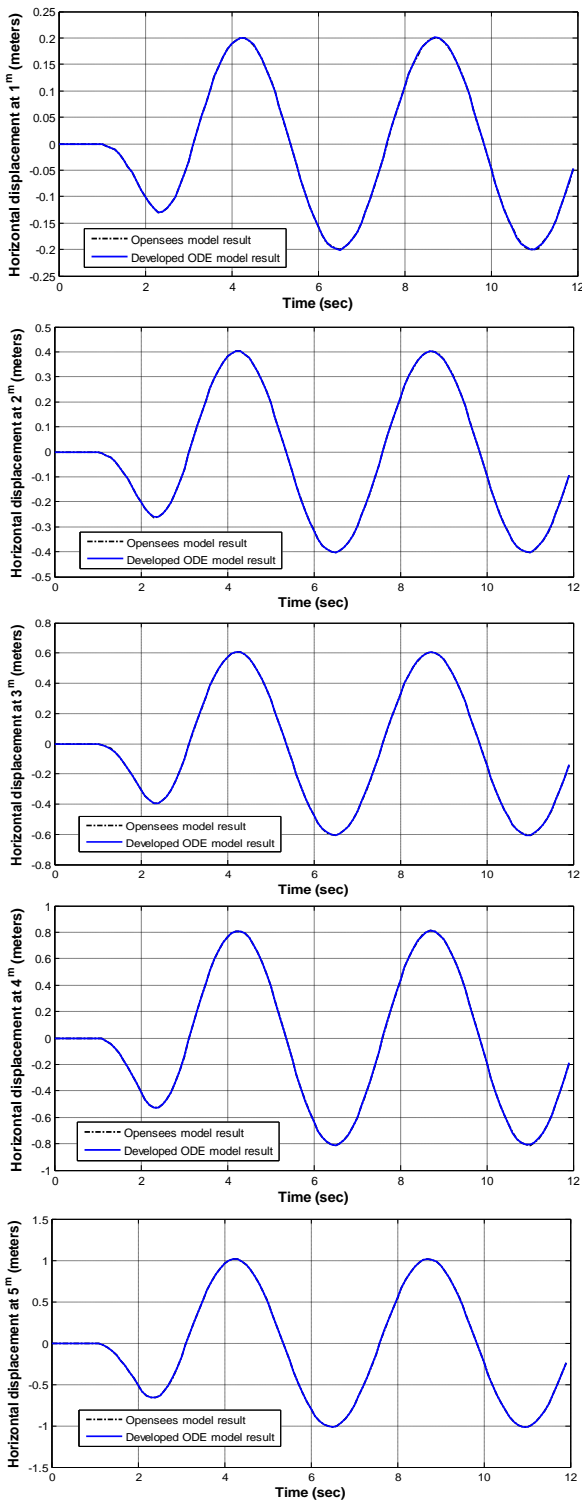


Figure6: The comparison of the simulation results using the OpenSees model and developed model. The horizontal displacements of five points, located on the cable at 1m, 2m, 3m, 4m and 5m, around vertical axis for a crane system with 50kg payload

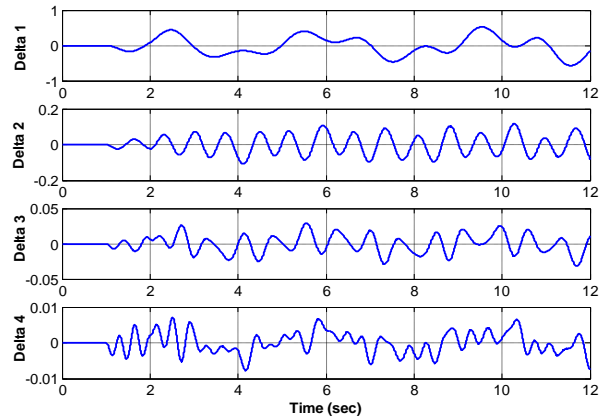


Figure4: The time histories of the amplitudes of the first four vibration modes of the same crane system with a 2kg payload under the exciting input force

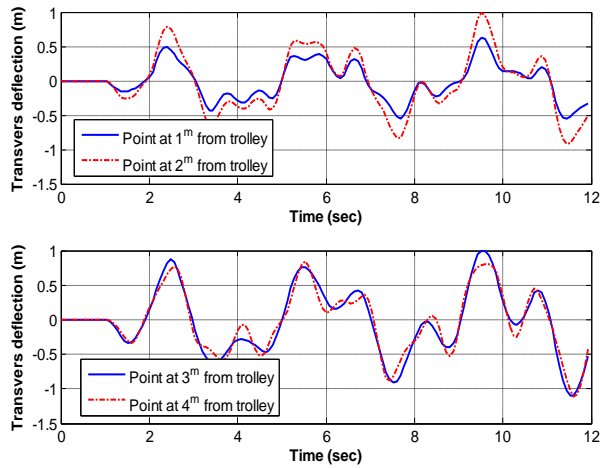


Figure5: Time histories of transverse deflections of the four selected points along the cable for the same crane system with a 2kg payload under the exciting force

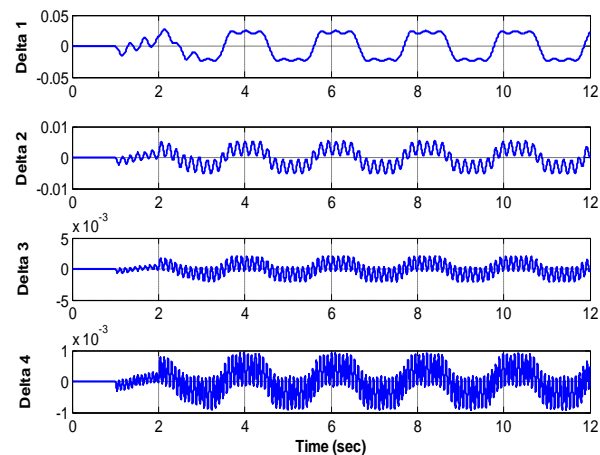


Figure6: The time histories of the amplitudes of the first four vibration modes of the same crane system with a 50kg payload under the exciting input force

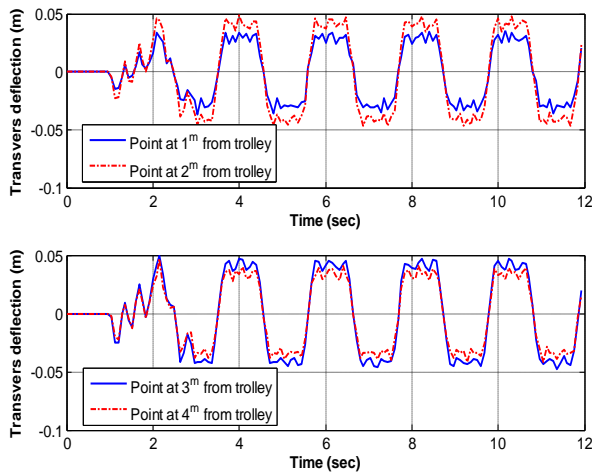


Figure7: Time histories of transverse deflections of the four selected points along the cable for the same crane system with a 50kg payload under the exciting input force

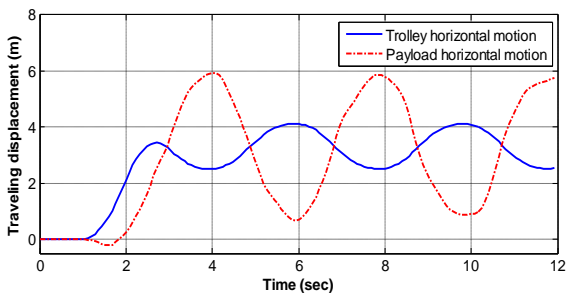


Figure8: Time histories of horizontal displacements of the trolley and the payload for the same crane system with a 2kg payload under the exciting input force

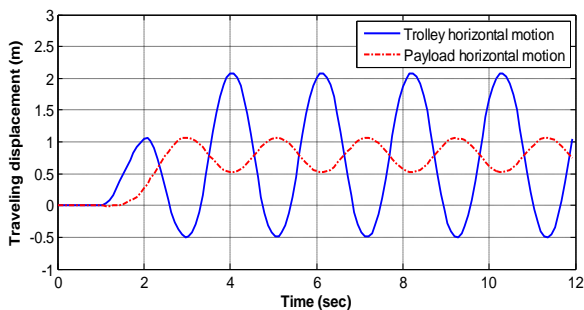


Figure9: Time histories of horizontal displacements of the trolley and the payload for the same crane system with a 50kg payload under the exciting input force

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