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Non-Linear Equation by using False Position Method

Mrs. Tasleem Sultana[1], Dr.Jyothi Shinde
[1]Royal Institute of Technology and Science
[2]Al- Habeeb College of Engineering & Technology, Chevella.

Abstract : In this paper we making a bookshelf to carry books. the material is wood having a young's modulus to find the maximum vertical deflection of the bookshelf. we can directly use this false position method because it converge the root values quickly when compared to bisection method.The advantage of the bisection method is its reliability but the disadvantage of the bisection method is it takes number of iterations to bisect to the optimum value as the interval to bisect will decrease so it takes more amount of time to get optimum value.

1.INTRODUCTION

Outline: The false position method is one of the numerical analysis method which is used to find the root of a non- linear equation the root of the equation is a point where the curve cuts X-axis when y=0 (1)

Method: The converges process in the bisection method is very slow. IT depends only on the choice of end points of the interval (a, b). The function f(x) does not have any role in finding the point c. Which is just the midpoint of a and b

It is used only to decide the next smaller interval (a, c)or (c ,b) (2)

A better approximation to c can be obtained by taking the straight line joining the points (a, f(a)) and (b, f(b)) intersecting the X- axis to obtain the value of c we can equate the two expressions of the slope m of the line.

$$x = \frac{af(b)-bf(a)}{f(b)-f(a)} \quad (3)$$

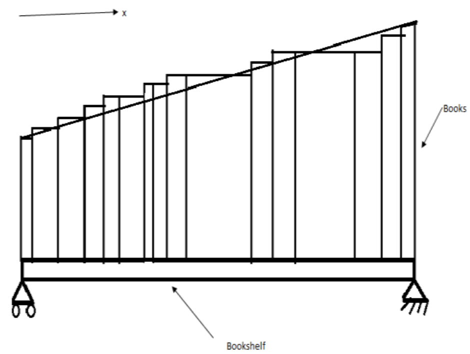
now the next smaller interval which brackets the root can be obtained by checking f(x)<0 or f(b)>0 selecting c by the above expression is called regular falsi method or false position method .

Result : Bi-Section Method V/s False position method .

Question: you are making a bookshelf to carry books that range from $8\frac{1}{2}$"to 11" in height and would take up 29" of space along the length. the material is wood having a Young's Modulus of 3.667Msi, thickness of $\frac{3}{8}$" and width of 12" you want to find the maximum vertical deflection of the bookshelf the vertical deflection of the self is given by

$$v(x) = 0.42493 \times 10^{-4}x^3 - 0.13533 \times 10^{-8}x^5 - 0.66722 \times 10^{-6}x^4 - 0.018507x$$

where x is the position along the length of the beam hence to find the maximum deflection we need to find where $f(x) = \frac{dv}{dx} = 0$ and conduct the second derivative test.



The equation that gives the position x where the deflection is maximum is given by

$$-0.67665 \times 10^{-8}x^4 - 0.26689 \times 10^{-5}x^3 + 0.12748 \times 10^{-3}x^2 - 0.018507=0$$

find the position x where the deflection is maximum conduct 3 iterations to estimate the root of the above equation find the absolute relative approximate error at the end of each iteration and the number of significant digits at least correct at the end of each iteration

Solution by Bisection Method :

$$-0.67665 \times 10^{-8}x^4 - 0.26689 \times 10^{-5}x^3 + 0.12748 \times 10^{-3}x^2 - 0.018507=0$$

$$-0.0000000067665\,x^4 - 0.0000026689\,x^3 + 0.00012748\,x^2 - 0.018507=0$$

f(0) =-0.018507<0

f(29) = 0.018826>0

Formula :

$$x = \frac{a+b}{2}$$

$$|E_a| = \left|\frac{x^{new} - x^{old}}{x^{new}}\right| \times 100$$

1. TABULAR:

S. no	a	b	x	F(x)	$\lvert E_a \rvert$
1	0	29	14.5	-0.0001399<0	-------------

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2	14.5	29	21.75	0.012824 >0	33.33
3	14.5	21.75	18.125	0.00675>0	20
4	14.5	18.125	16.313	0.003352 >0	11.11
5	14.5	16.313	15.407	0.001612 >0	5.8804
6	14.5	15.407	14.954	0.0007192>0	3.0293
7	14.5	14.954	14.727	0.00029853>0	1.5414
8	14.5	14.727	14.614	0.00008025>0	0.773
9	14.5	14.614	14.557	-0.00002984<0	0.392
10	14.557	14.614	14.586	0.00002617>0	0.199
11	14.557	14.586	14.572	-0.000000873<0	0.0961
12	14.572	14.586	14.579	0.00001265>0	0.0480

By the 12^{th} iteration $|E_a|$ = 0.0480

$$|E_a| \leq 0.5 \times 10^{2-m}$$

$$0.0480 \leq 0.5 \times 10^{2-m}$$

dividing the above equation by 0.5

0.096$\leq 10^{2-m}$

$\log(0.096) \leq 2 - m$

$m \leq 2 - \log(0.096)$

$m = 3.02$ approximate

By the 11^{th} iteration $|E_a|$ = 0.0961

$|E_a| \leq 0.5 \times 10^{2-m}$

$0.0961 \leq 0.5 \times 10^{2-m}$

dividing the above equation by 0.5

0.1922$\leq 10^{2-m}$

$\log(0.1922) \leq 2 - m$

$m \leq 2 - \log(0.1922)$

$m = 2.716$ approximate

By the 10^{th} iteration $|E_a|$ = 0.199

$|E_a| \leq 0.5 \times 10^{2-m}$

0.199 $\leq 0.5 \times 10^{2-m}$

dividing the above equation by 0.5

0.398$\leq 10^{2-m}$

$\log(0.398) \leq 2 - m$

$m \leq 2 - \log(0.398)$

$m = 2.40012$ approximate

therefore m=2

The number of significant digits at least correct in the estimated

root is 14.586 is 2

Solution by False Position method:

$-0.67665 \times 10^{-8}x^4 - 0.26689 \times 10^{-5}x^3 + 0.12748 \times 10^{-3}x^2 - 0.018507$=0

$-0.0000000067665\,x^4 - 0.0000026689\,x^3 + 0.00012748\,x^2 - 0.018507$=0

f(0) =-0.018507<0

f(29) = 0.018826>0

Formula :

$$x = \frac{af(b) - bf(a)}{f(b) - f(a)}$$

$$|E_a| = \left|\frac{x^{new} - x^{old}}{x^{new}}\right| \times 100$$

Here a=0 b=29

$x_1 = \frac{0f(29)-29f(0)}{f(29)-f(0)}$ =14.376

$f(x_1) = f(14.376) = -0.0003793 < 0$

Since $f(x_1) < 0$ and $f(b) > 0$

therefore

$a = x_1 = 14.376$ and $b = 29$

$x_2 = \frac{14.376f(29)-29f(14.376)}{f(29)-f(14.376)}$ =14.665

$f(x_2) = f(14.665) = 0.0001788 > 0$

$$|E_a| = \left|\frac{x^{new} - x^{old}}{x^{new}}\right| \times 100 \quad = \left|\frac{14.665 - 14.376}{14.665}\right| \times 100$$

=1.97%

Since $f(x_2) > 0$ and $f(a) < 0$

therefore

a=14.376 and b=$x_2 = 14.665$

$x_3 = \frac{14.376f(14.665) - 14.665f(14.376)}{f(14.665) - f(14.376)}$ =14.572

$f(x_3) = f(14.572) = -0.00000087 < 0$

$|E_a| = \left|\frac{x^{new} - x^{old}}{x^{new}}\right| \times 100$

$x^{old} = 14.665$ and $x^{new} = 14.572$

$= \left|\frac{14.572 - 14.665}{14.572}\right| \times 100$

=0.638%

Since $f(x_3) < 0$ and $f(b) > 0$

therefore

a=x_3 =14.572 and b= 14.665

$x_4 = \frac{14.572f(14.665) - 14.665f(14.572)}{f(14.665) - f(14.572)}$ =14.572

$f(x_4) = f(14.572) = -0.00000087 < 0$

The above data Tabular form

S. no	a	b	x	F(x)	$\|E_a\|$
1	0	29	14.376	-0.0003793<0	--------------
2	14.376	29	14.665	0.0001788>0	1.97 %
3	14.376	14.665	14.572	-0.00000087 <0	0.638 %
4	14.572	14.665	14.572	-0.00000087 <0	0%

After the 3rd iteration the value of x is not changing it is constant

At the end of the 3rd iteration $|E_a|$ = 0.638%

Hence the number of significant digits at least correct is given by largest value of m for

which

$|E_a| \leq 0.5 \times 10^{2-m}$

$0.638 \leq 0.5 \times 10^{2-m}$
dividing the above equation by 0.5
$1.276 \leq 10^{2-m}$

$\log(1.276) \leq 2 - m$

$m \leq 2 - \log(1.276)$

$m = 1.894$ approximate

therefore m=2

The number of significant digits at least correct in the estimated

root is 14.572 is 2

CONCLUSION:

false position method usually converges more rapidly than Bisection method.

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