

Fractional Order Filter Designing Using Modified Genetic Algorithm

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ABSTRACT : A filter is an electrical network that alters the amplitude and/or phase characteristics of a signal with respect to frequency. Digital filter provide an important role in world of communication and computation. Fractional order Low pass Butterworth filters characteristics have been investigated in this paper, which cannot be achieved with conventional integer order Butterworth filters. In this paper, a modified version of genetic algorithm has been applied to design fractional order Butterworth filter. The designed filter provide better performance parameters as compared to classical integer order based design

Keywords : Digital filter, Fractional order, genetic algorithm, Butterworth filter, evolutionary optimization, Amplitude response, phase response.

I. INTRODUCTION

Ideally, a filter will not add new frequencies to the input signal, nor will it change the component frequencies of that signal, but it will change the relative amplitudes of the various frequency components and/or their phase relationships. Filters are often used in electronic systems to emphasize signals in certain frequency ranges and reject signals in other frequency ranges. There are two types of filter: analog and digital. FIR Filter is the kind of digital filter, which can be used to perform all kinds of filtering[1]. A digital filter uses a digital processor to perform numerical calculations on sampled values of the signal. The processor may be a general purpose computer such as a PC, or a specialized DSP (Digital Signal Processor) chip. Digital filters are used in a wide variety of signal processing applications, such as spectrum analysis, digital image processing, and pattern recognition. Digital filters eliminate a number of problems associated with their classical analog counterparts and thus are preferably used in place of analog filters. The analog input signal must first be sampled and digitized using an ADC (analog to digital converter). The resulting binary numbers, representing successive sampled values of the input signal, are transferred to the processor, which carries out numerical calculations on them. Fast DSP processors can handle complex combinations of filters in parallel or cascade (series), making the hardware requirements relatively simple and compact in comparison with the equivalent analog circuitry. The following list gives some of the main advantages of digital over analog filters.

A digital filter is programmable, i.e. its operation is determined by a program stored in the processor's memory. This means the digital filter can easily be changed without affecting the circuitry (hardware). An analog filter can only be changed by redesigning the filter circuit. Digital filters are easily designed, tested and

implemented on a general purpose computer or workstation. The characteristics of analog filter circuits (particularly those containing active components) are subject to drift and are dependent on temperature. Digital filters do not suffer from these problems, and so are extremely stable with respect to both time and temperature.

Unlike their analog counterparts, digital filters can handle low frequency signals accurately. As the speed of DSP technology continues to increase, digital filters are being applied to high frequency signals in the RF (radio frequency) domain, which in the past was the exclusive preserve of analog technology. Digital filters are very much more versatile in their ability to process signals in a variety of ways; this includes the ability of some types of digital filter to adapt to changes in the characteristics of the signal.

Fast DSP processors can handle complex combinations of filters in parallel or cascade (series), making the hardware requirements relatively simple and compact in comparison with the equivalent analog circuitry. Traditional filter classification begins with classifying a filter according to its impulse response. These terms refer to the differing "impulse responses" of the two types of filter. Digital filter can be classified as one of the following types: Finite impulse response (FIR) filter, also known as non recursive filters (in a non recursive filter the current output is calculated solely from the current and previous input values). Infinite impulse response (IIR) filter, also known as recursive filter (a recursive filter is one which in addition to input values also uses previous output values).

II. COMPARISION OF IIR AND FIR FILTERS

Because designing digital filters involves making compromises to emphasize a desirable filter characteristic over a less desirable characteristic, comparing FIR and IIR filters can help in selecting the appropriate filter design for a particular application. IIR filters have the advantages of providing the higher selectivity for a particular order.

IIR filters can achieve the same level of attenuation as FIR filters but with far fewer coefficients. Therefore, an IIR filter can provide a significantly faster and more efficient filtering operation than an FIR filter. FIR filters provide a linear phase response. IIR filters provide a nonlinear phase response. FIR filters are used for applications that require linear phase responses like high quality audio systems. IIR filters are used for applications that do not require phase information, such as signal monitoring applications.

Compared to IIR filters, FIR filters sometimes have the disadvantage that they require more memory and/or calculation

to achieve a given filter response characteristic. Also, certain responses are not practical to implement with FIR filters. FIR filters are always stable because they are implemented using an all zero transfer function. Since no poles can fall outside the unit circle, the filter will always be stable. But because of this, the order of FIR filter is much higher than the IIR filter which has the comparable magnitude response. The higher order of the FIR filters lead to longer processing times and larger memory requirements.

III. FILTER DESIGN TECHNIQUES

The main FIR filter design essentially consists of two parts:

- i. Approximation problem
- ii. Realization problem.

The approximation stage takes the specification and gives transfer function. Realization part deals with choosing the structure to implement the transfer function which maybe in the form of circuit diagram or in the form of a program.

There are various methods to design FIR filter as follows:

- I. Window technique.
- II. The frequency sampling technique.
- III. Optimal filter design methods.

IV. WINDOW TECHNIQUE

The windowing design technique is simple and convenient but not optimal i.e. order achieving is not minimum possible. Some of the windows commonly used are Blackman, Blackman-harris, Kaiser, Bohman, Chebyshev, Flat top, Gaussian, Hamming, Hann, Parzen, Rectangular, etc.

In frequency sampling technique for FIR filter design we specify the desired frequency response $H(\omega)$ at a set of equally spaced frequencies at N samples. This method is useful for the design of non prototype filters where the desired magnitude response can take any irregular shape. The various optimal filter design methods are Least square, Equiripple, Maximally flat, Generalized equiripple, Constrained band equiripple, etc.

The several effects of windowing the Fourier coefficients of the filter on the result of the frequency response of the filter are as follows:

- (i) A major effect is that discontinuities in $H(\omega)$ become transition bands between value on either side of the discontinuity.
- (ii) The width of the transition bands depends on the width of the main lobe of the frequency response of the window function, $W(\omega)$ i.e. $W(\omega)$.
- (iii) Since the filter frequency response is obtained via a convolution relation, it is clear that the resulting filters are never optimal in any sense.
- (iv) As M (the length of the window function) increases, the main lobe width of $W(\omega)$ is reduced which reduces the width of the transition band, but this also introduces more ripple in the frequency response.

- (v) The window function eliminates the ringing effects at the band edge and does result in lower side lobes at the expense of an increase in the width of the transition band of the filter.

Some of the windows commonly used are as follows:

1. Bartlett triangular window.
2. Generalized cosine windows (Rectangular, Hanning, Hamming and Blackman).
3. Kaiser window with parameter β .

The Bartlett window reduces the overshoot in the designed filter but spreads the transition region considerably. The Hamming and Blackman windows use progressively more complicated cosine functions to provide a smooth truncation of the ideal impulse response and a frequency response that looks better. The best window results probably come from using the Kaiser window, which has parameter β that allows adjustment of the compromise between the overshoot reduction and transition region width spreading.

Advantages of window method:

The major advantages of using window method are their relative simplicity as compared to other methods and ease of use. The fact that well defined equations are often available for calculating the window coefficients has made this method successful. There are following problems in filter design using window method: (i) This method is applicable only if $H(\omega)$ is absolutely integrable i.e only if $\int_{-\infty}^{\infty} H(\omega) d\omega$ can be evaluated. When $H(\omega)$ is complicated or cannot easily be put into a closed form mathematical expression, evaluation of $H(\omega)$ becomes difficult. (ii) The use of windows offers very little design flexibility e.g. in low pass filter design, the passband edge frequency generally cannot be specified exactly since the window smears the discontinuity in frequency. Thus the ideal LPF with cut-off frequency f_c , is smeared by the window to give a frequency response with passband response with passband cutoff frequency f_1 and stopband cut-off frequency f_2 . (iii) Window method is basically useful for design of prototype filters like lowpass, highpass, bandpass etc. This makes its use in speech and image processing applications very limited.

V. THE FREQUENCY SAMPLING TECHNIQUE

In this method, the desired frequency response is provided as in the previous method. Now the given frequency response is sampled at a set of equally spaced frequencies to obtain N samples. Thus, sampling the continuous frequency response $H(\omega)$ at N points essentially gives us the N -point DFT of $H_d(2\pi nk/N-s)$. The steps involved in this method suggested by Rabiner are as follows:

- (i) The desired magnitude response is provided along with the number of samples, N . Given N , the designer determines how fine an interpolation will be used.
- (ii) It was found by Rabiner that for designs they investigated, where N varied from 15 to 256, $16N$ samples of $H(\omega)$ lead to reliable computations, so 16 to 1 interpolation was used.

(iii) Given N values of H_k , the unit sample response of filter to be designed, $h(n)$ is calculated using the inverse FFT algorithm.

(iv) In order to obtain values of the interpolated frequency response two procedures were suggested by Rabiner. They are
(a) $h(n)$ is rotated by $N/2$ samples (N even) or $(N-1)/2$ samples for N odd to remove the sharp edges of impulse response, and then $15N$ zero-valued samples are symmetrically placed around the impulse response. (b) $h(n)$ is split around the $N/2$ nd sample, and $15N$ zero-valued samples are placed between the two pieces of the impulse response.

(v) The zero augmented sequences are transformed using the FFT algorithm to give the interpolated frequency responses.

Merits and demerits of frequency sampling technique:

(i) Unlike the window method, this technique can be used for any given magnitude response.

(ii) This method is useful for the design of non-prototype filters where the desired magnitude response can take any irregular shape.

There are some disadvantages with this method i.e the frequency response obtained by interpolation is equal to the desired frequency response only at the sampled points. At the other points, there will be a finite error present.

VI. OPTIMAL FILTER DESIGN METHODS

Wherever Many methods are present under this category. The basic idea in each method is to design the filter coefficients again and again until a particular error is minimized. The various methods are as follows:

- (i) Least squared error frequency domain design[2]
- (ii) Weighted Chebyshev approximation
- (iii) Nonlinear equation solution for maximal ripple FIR filters.
- (iv) Polynomial interpolation solution for maximal ripple FIR filters.

These are explained as follows:

(i) Least squared error frequency domain design:

As seen in the previous method of frequency sampling technique there is no constraint on the response between the sample points, and poor results may be obtained. The frequency sampling technique is more of an interpolation method rather than an approximation method. This method controls the response between the sample points by considering a number of sample points larger than the order of the filter. The purpose of most filters is to separate desired signals from undesired signals or noise. As the energy of the signal is related to the square of

the signal, a squared error approximation criterion is appropriate to optimize the design of the FIR filters.

(ii) Weighted Chebyshev Approximation[3], [4] :

In this method, following terms are defined :

$H_d(w)$ = the desired (real) frequency response of the filter
 $H(\omega)$ = the frequency response of the designed filter

$W(\omega)$ = the frequency response of the weighting function .

The weighting function enables the designer to choose the relative size of the error in different frequency bands. The frequency response of linear phase filters for four different types can be written as follows:

$$H(\omega) = \sum_{n=0}^{(N-1)/2} a(n) e^{-j\omega n} H(\omega) \quad (1)$$

(iii) Nonlinear Equation solution for maximal ripple FIR filters:

The real part of the frequency response of the designed FIR filter can be written as $a(n)\cos(\omega n)$ [Rab75] where limits of summation and $a(n)$ vary according to the type of the filter. The number of frequencies at which $H(\omega)$ could attain an extreme is strictly a function of the type of the linear phase filter i.e. whether length N of filter is odd or even or filter is symmetric or anti-symmetric. At each extreme, the value of $H(\omega)$ is predetermined by a combination of the weighting function $W(\omega)$, the desired frequency response, and a quantity that represents the peak error of approximation distributing the frequencies at which $H(\omega)$ attains an extreme value among the different frequency bands over which a desired response was being approximated. Since these filters have the maximum number of ripples, they are called maximal ripple filters.

(iv) Polynomial Interpolation Solution for Maximal Ripple FIR filters:

This algorithm is basically an iterative technique for producing a polynomial $H(\omega)$ that has extreme of desired values. The algorithm begins by making an initial estimate of the frequencies at which the extreme in $H(\omega)$ will occur and then uses the well-known Lagrange interpolation formula to obtain a polynomial that alternatively goes through the maximum allowable ripple values at these frequencies. It has been experimentally found that the initial guess of extreme frequencies does not affect the ultimate convergence of the algorithm but instead affects the number of iterations required to achieve the desired result.

VII. FRACTIONAL ORDER FILTER DESIGN USING GENETIC ALGORITHM

The flow of genetic algorithm can be summarized as: Initially a population is created using random numbers within a minimum and maximum limit. The individuals in the population are then

evaluated using objective function. The objective function provided is given in (2). Fitness of the objective function gives the individuals a score based on how well they perform at the given task. Two individuals are then selected based on their fitness, the higher the fitness, the higher the chance of being selected. These individuals then create one or more offspring, after which the offspring are mutated randomly. This continues until a suitable solution has been found or a certain number of generations have passed, depending on the needs of the programmer[1], [5].

VIII. RESULTS AND DISCUSSION

Example - Suppose Given specification for low pass Butterworth filter is :-

- (a) Pass Band Frequency (Ω_p) = 100 Hz
- (b) Stop Band Frequency (Ω_s) = 300 Hz
- (c) Pass Band Attenuation (α_p) = 1dB
- (d) Stop Band Attenuation (α_s) = 12dB

Order of the filter (N)

$N=1.8428$ (truncating upto 1 decimal place)

$N=1.8$

It means $N=1$ (integer part) + 0.8 (fractional part)

Transfer function $G(s)$ of LPFB for $N=1.8$

$$4.9613e+007$$

$$s^{2.6} - 0.016796s^{2.2} - 2.8422e-014s^2 - 0.37818s^{1.8} + 489.16s^{1.6} - 5.349s^{1.4} - \dots + 4.9613e+007$$

Comparative Frequency response of $G(s)$ for $N = 1, 2$

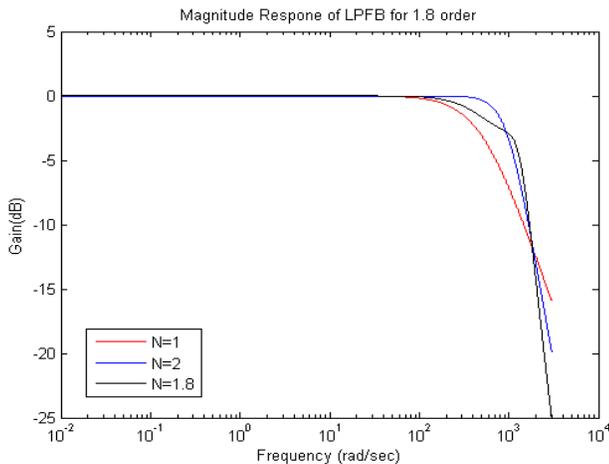


Fig 1. Frequency response of the fractional order filter designing using genetic algorithm

IX. Conclusion

The paper details the classical techniques involved in the design of filters and genetic algorithm to design fractional order filter. Every method has its own advantages and disadvantages and is selected depending on the type of filter to be designed. The window method is basically used for the design of prototype filters like the low-pass, high-pass, band-pass etc. They are not very suitable for designing of filters with any given frequency response. On the other hand, the frequency sampling technique is suitable for designing of filters with a given magnitude response. The ideal frequency response of the filter is approximated by placing appropriate frequency samples in the z - plane and then calculating the filter co-efficient using the IFFT algorithm. The best method is optimal filter design methods using modified genetic algorithm. It gives more accurate and precise results as compared to classical filter design approaches. It has least pass band ripples and the design is good for less order filters.

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